

Permutations

If X is a set, a bijection $f : X \rightarrow X$ is called a *permutation* of X

X

$$f: \mathbb{Z} \rightarrow \mathbb{Z} \quad f(x) = x+1$$

$$X = \{1, 2\} \quad f(1) = 2 \quad f(2) = 1$$

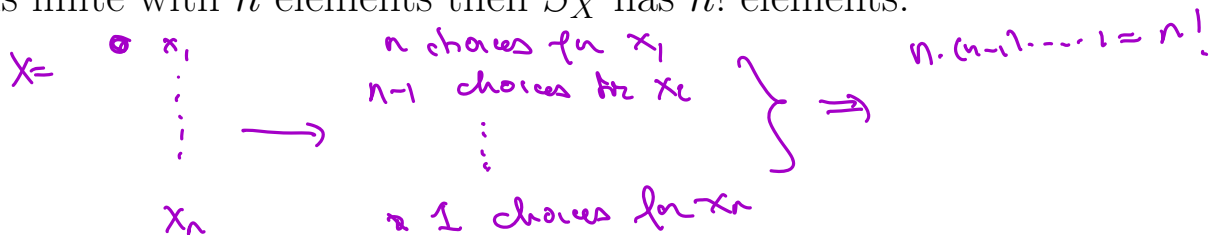
The permutations of a set form a group S_X under composition of functions, with identity element $id_X : X \rightarrow X$.

$$id_X(x) = x$$

$$f(g \circ h)(x) = f(g(h(x))) = (f \circ g) \circ h(x)$$

f bijective $\Rightarrow f$ has an inverse function

If X is finite with n elements then S_X has $n!$ elements.



If X is finite with n elements we can assume

$$X = \{1, 2, \dots, n\}.$$

The permutations of this set X is called the symmetric group on n elements and written S_n . $n!$ elements.

Multiplication of Permutations

An element $\sigma \in S_n$ sending $\sigma(i) = x_i$ can be written

$$\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ x_1 & x_2 & \cdots & x_n \end{pmatrix}$$

$\{\dots, x\}$
 $\sigma(1) = x_1$
 $\sigma(2) = x_2$
 $\sigma(3) = x_3$
 \vdots
 $\{1, \dots, n\}$

Multiplication is *composition of functions* and goes *right to left*.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$$

$\tau(1) = 2$
 $\tau(2) = 3$
 $\sigma(2) = 1$
 $\sigma(3) = 4$

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \Rightarrow \sigma^{-1}$$

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$