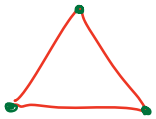
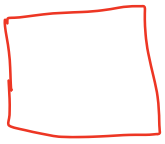
Dihedral Groups

D_n is the group of symmetries of a regular n-gon. $(n \geq 3)$

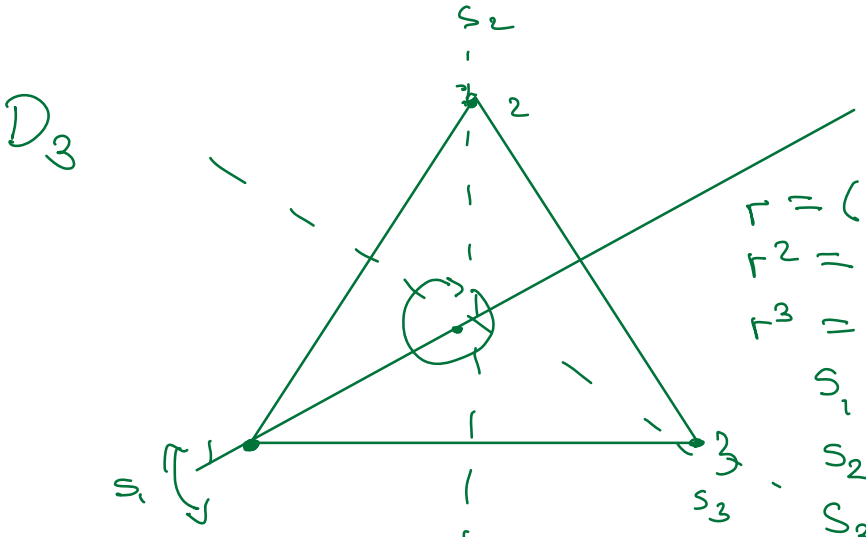
D_3



D_4

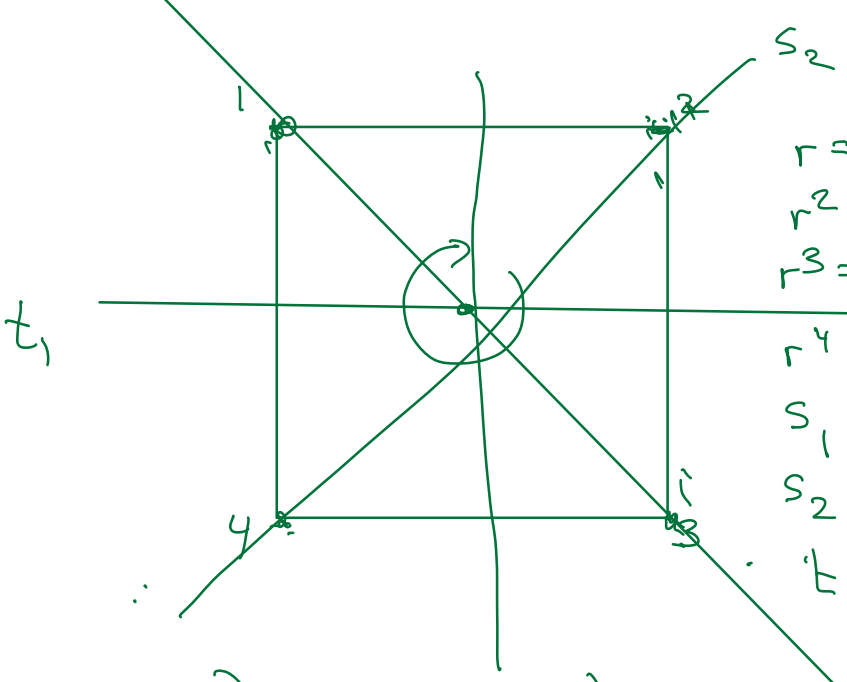


D_3



$r = (123)$.
$r^2 = (132)$.
$r^3 = e$.
$s_1 = (23)$.
$s_2 = (13)$.
s_3 (12).

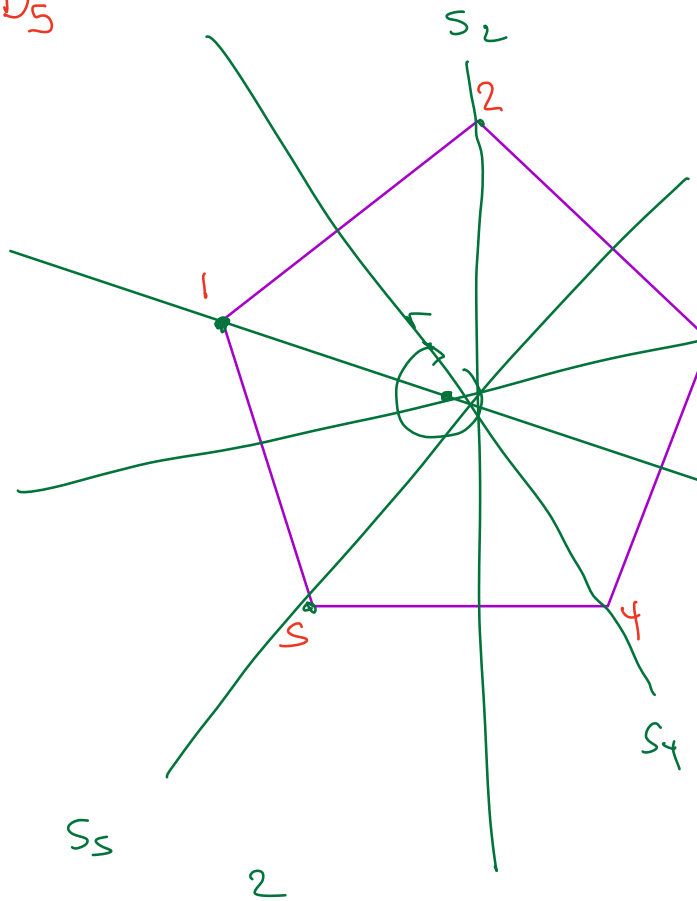
$D_3 = \langle s_3 \rangle$ $|D_3| = 6$.



$r = (1234)$.
$r^2 = (13)(24)$.
$r^3 = (1432)$.
$r^4 = e$.
$s_1 = (24)$.
$s_2 = (13)$.
$t_1 = (14)(23)$.
$t_2 = (12)(34)$.

$|D_4| = 8$ elts $\neq S_4$

D_5



$r = (1\,2\,3\,4\,5)$

$r^2, r^3, r^4, r^5 = e$

5 rotations

5 reflections

10 elements.

$s_1 = (2\,5)(3\,4)$

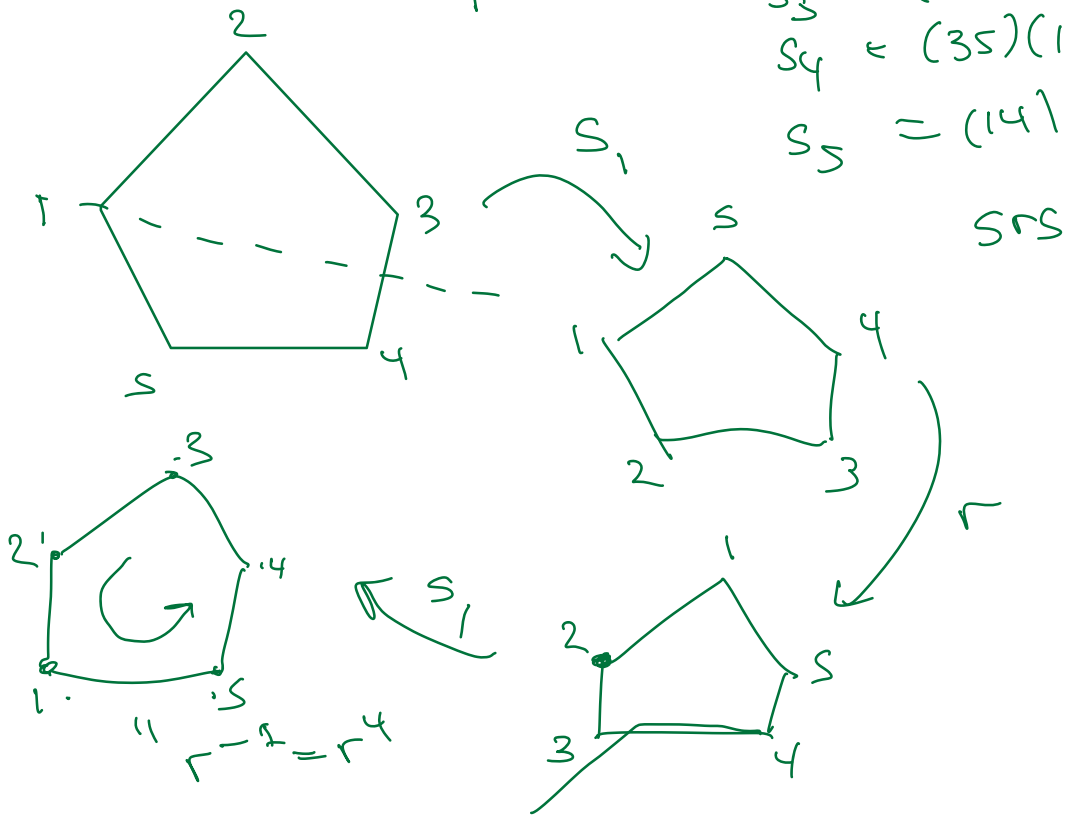
$s_2 = (1\,3)(5\,4)$

$s_3 = (2\,4)(1\,5)$

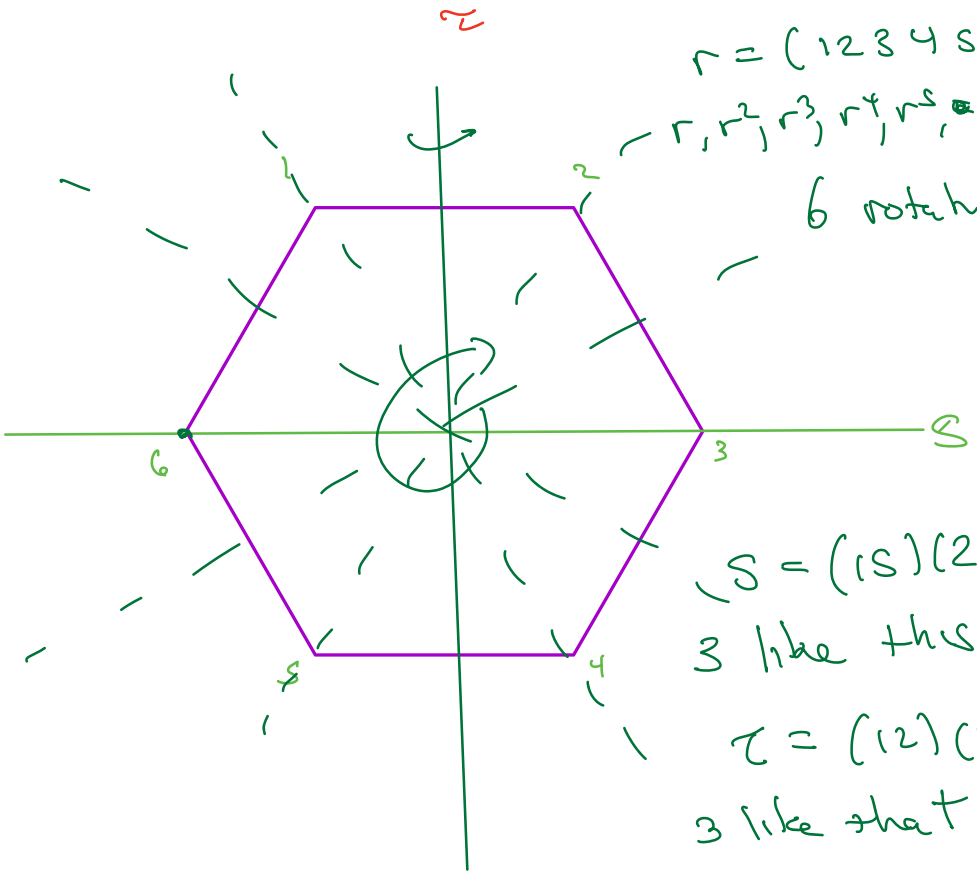
$s_4 = (3\,5)(1\,2)$

$s_5 = (1\,4)(2\,3)$

$srs = r^{-1}$



D_6



$r = (1\,2\,3\,4\,5\,6)$

$r, r^2, r^3, r^4, r^5, r^6 = e$

6 rotations

$S = (1\,5)(2\,4)$

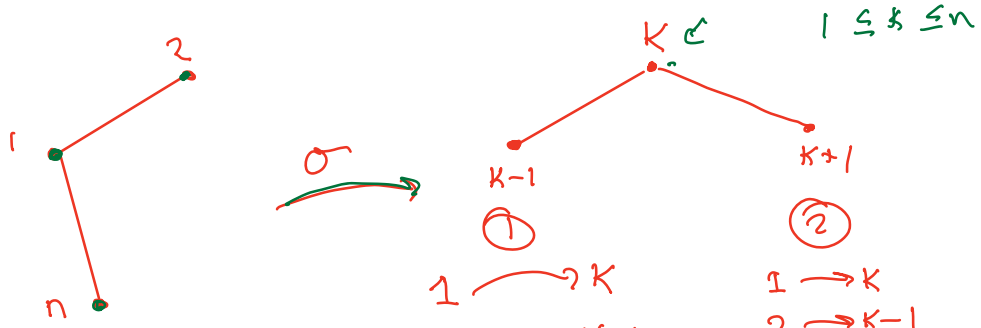
3 like this ✓

$\tau = (1\,2)(3\,6)(4\,5)$

3 like that ✓

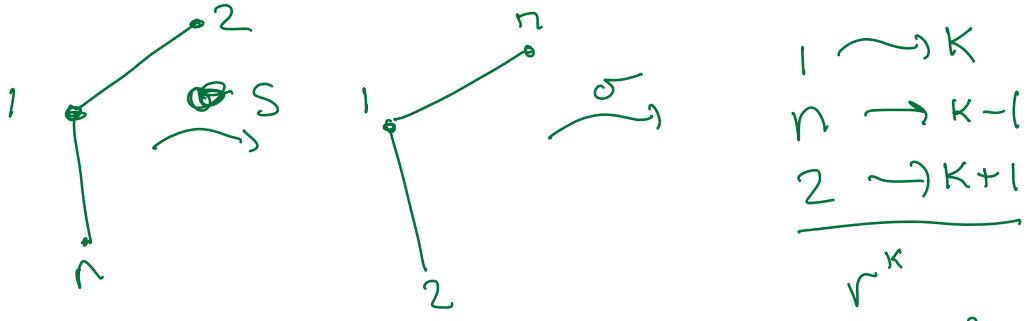
12 elements

$|D_6| = 12 \leq S_6 \quad 6! = 720$



$r = (1\,2\,3\,4 \cdots n)$

S be reflection fixing 2.



$\sigma S = r^k$ $\quad S^2 = e$

$\sigma S^2 = r^k S$ $\quad \sigma = r^k S$

Every elt in D_n is either r^k $\quad k = 0, \ldots, n-1$

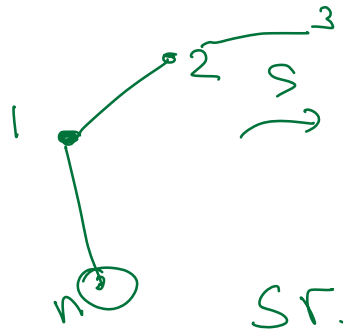
$r^n = e$

OR $r^k S$ $\quad k = 0, \ldots, n-1$

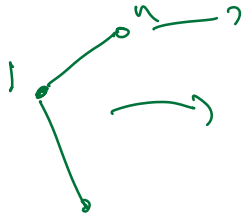
S = reflection fixing 1.

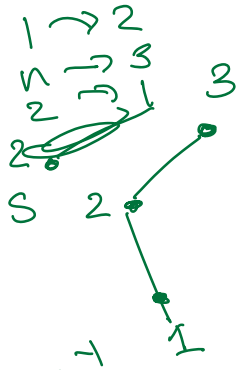
$|D_n| = 2n$

srs



s





$srs = r^{-1}$ 2

$sr = r^{-1}s$

r^k

$r^k s$

$r^k \cdot r^j s = r^{k+j} s$

$srs = r^{-1}$

$sr = r^{-1}s$

$r^k s r^j = r^k r^{-j} s = r^{k-j} s$

$r^k s r^j s = r^{k-j} s^2 = r^{k-j}$

$r^n = e \quad s^2 = e \quad \& \quad srs = r^{-1}$

presentation of the group.