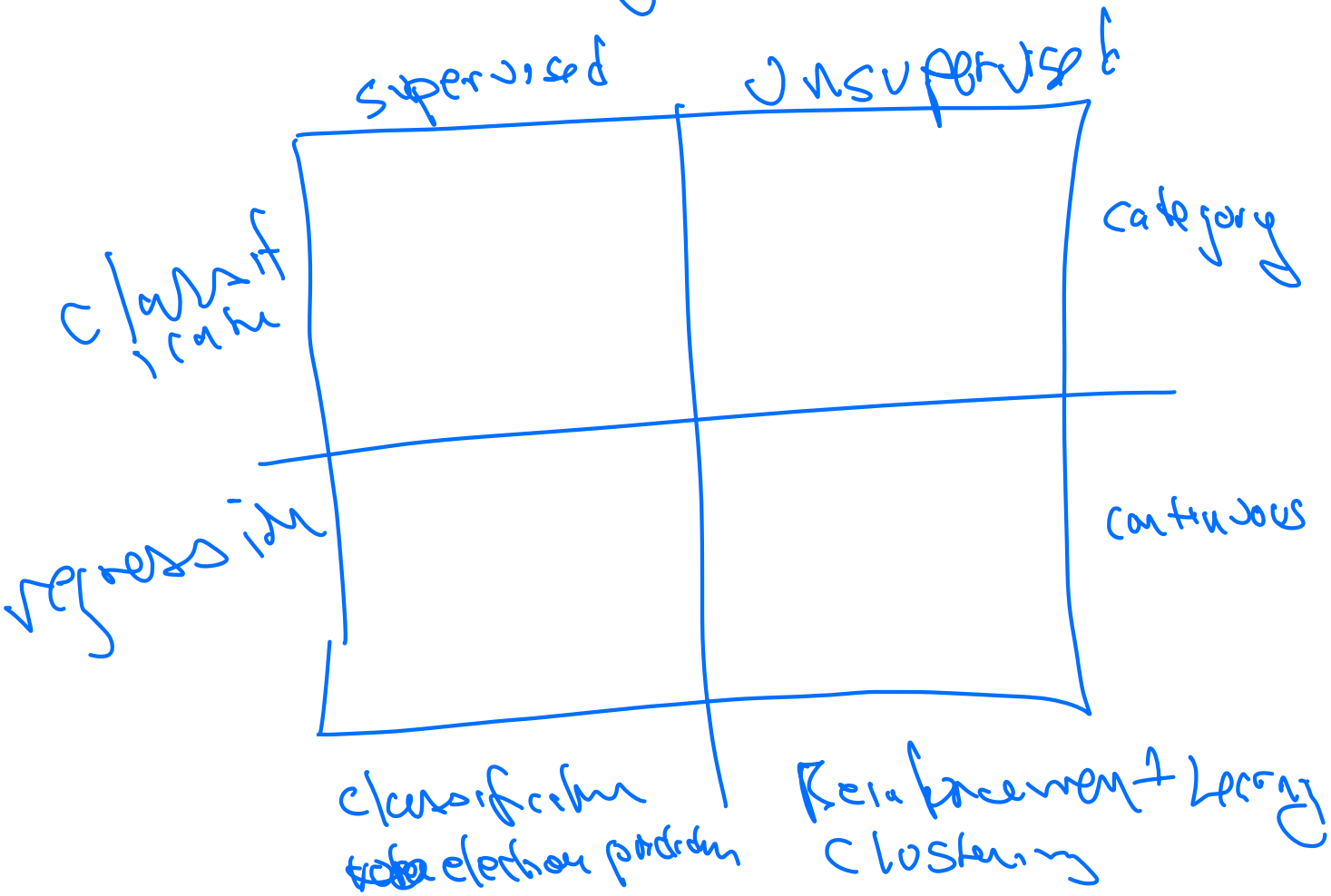
Math 3180

Spring 2026

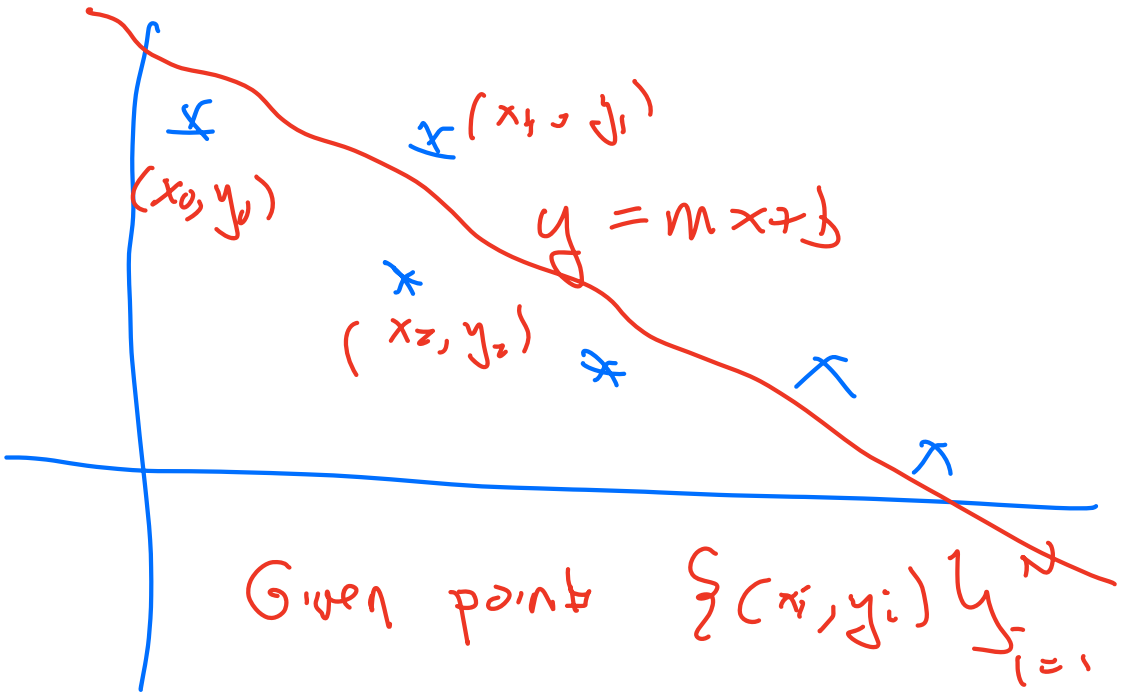
Vernor Vinge

Measured in Realtime



Supervised:

data	source	→	target
	images	→	labelled cat or not
	demographic info	→	voting history



Given pairs $\{(x_i, y_i)\}_{i=1}^{N}$

Have $y = mx+b$

for each x_i, this predicts $y_i^* = mx_i+b \longleftrightarrow$ "true" y_i.

Measure how good these predictions are.

Mean Squared Error $= \frac{1}{N}\sum_i (y_i^* - y_i)^2$ } LOSS

$$l' = \frac{1}{N}\sum_i \left| y_i^* - y_i \right|$$ } Function

---

Given (x_i, y_i)

Want m, b so that

when $y_i^* = mx_i + b$

the $MSE = \frac{1}{N}\sum (y_i^* - y_i)^2$

is minimized.

$$L = MSE = \frac{1}{N}\sum_{i=1}^{N} (mx_i + b - y_i)^2$$

How?

$z = m^2 + b^2$

$z = m^2 + mb + b^2$

$\frac{\partial L}{\partial m}$ $\frac{\partial L}{\partial b}$

$$\frac{\partial L}{\partial m} = \frac{1}{N}\frac{\partial}{\partial m}\sum_{i=1}^{N} (mx_i + b - y_i)^2$$

$$= \frac{1}{N}\sum_{i=1}^{N} \frac{\partial}{\partial m}(mx_i + b - y_i)^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} 2(mx_i + b - y_i)\, x_i$$

$$= \frac{2}{N} \sum_{i=1}^{N} (mx_i^2 + bx_i - y_i x_i)$$

$$= \frac{2}{N} \left[m\sum x_i^2 + b\sum x_i - \sum x_i y_i \right]$$

$$S_{xx} = \sum x_i^2 \qquad S_x = \sum x_i \qquad S_{xy} = \sum x_i y_i$$

$$\frac{\partial L}{\partial m} = 0 \Rightarrow mS_{xx} + bS_x - S_{xy} = 0$$

$$\frac{\partial L}{\partial b} = \frac{1}{N} \frac{\partial}{\partial b} \sum (mx_i + b - y_i)^2$$

$$= \frac{1}{N} \sum \frac{\partial}{\partial b} (mx_i + b - y_i)^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} 2(mx_i + b - y_i)$$

$$= \frac{2}{N} \sum_{i=1}^{N} (mx_i + b - y_i)$$

$$= \frac{2}{N} \left[m\sum x_i + b \sum_{i=1}^{N} 1 - \sum y_i \right]$$

$$= \frac{2}{N} \left[mS_x + Nb - S_y \right] = 0$$

$$m S_{xx} + b S_x - S_{xy} = 0$$

$$m S_x + N b - S_y = 0$$

$$\boxed{\begin{aligned} m S_{xx} + b S_x &= S_{xy} \\ m S_x + N b &= S_y \end{aligned}}$$

What can go wrong here? (No solution? ∞ solutions?)