

Name: _____

Date: _____

Math 2710 — Exam 1

This exam has 15 problems or parts of problems. Each is worth 5 points for a total of 75 points. Write clearly and explain your answers.

1. What is the cardinality of the set

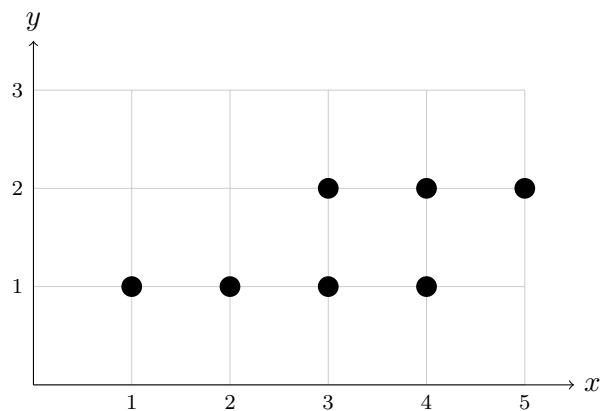
$$\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}?$$

2. Let $A = \{x \in \mathbb{R} : x^2 = 2\}$ and let $B = \{a, c, e\}$. List the elements of $A \times B$.

3. Let $A = \{1, 2, 3\}$. List the elements of $\{X \in \mathcal{P}(A) : |X| = 2\}$.

4. Let $A = \{0, 2, 4, 6, 8\}$ and $C = \{2, 4, 8\}$. What is $(A \times C) \cap (C \times C)$?

5. Let $X = \{1, 2, 3, 4, 5\}$ and let $Y = \{1, 2\}$. The shaded points in the graph below describe a set $Z \subseteq X \times Y$.



Decide whether each of the following is true, and give a brief explanation of your answer.

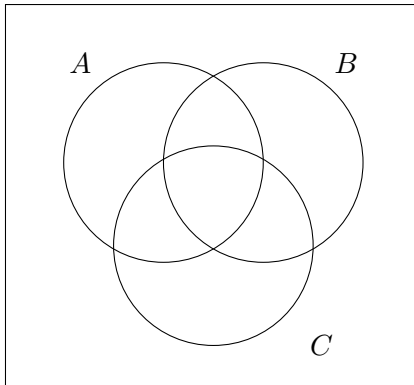
(a) $\forall x \in X, \forall y \in Y, (x, y) \in Z$.

(b) $\exists x \in X, \exists y \in Y, (x, y) \in Z$.

(c) $\forall x \in X, \exists y \in Y, (x, y) \in Z$.

(d) $\exists x \in X, \forall y \in Y, (x, y) \in Z$.

6. Shade the region of the Venn diagram below corresponding to $(A \cup B) - C$.



7. Write the following set in set-builder notation:

$$\{0, 1, 4, 9, 16, 25, \dots\}.$$

8. For $i = 1, 2, 3, \dots$, let

$$X_i = \{x \in \mathbb{R} : 2i \leq x \leq 2i + 1\}.$$

Draw a sketch of $\bigcup_{i \in \mathbb{N}} X_i$ on the number line.

9. Without changing the meaning, convert the following sentence into the form “If P , then Q .” Be sure to specify P and Q carefully.

A geometric series with ratio r converges if $|r| < 1$.

10. Convert the following into a sentence of the form “ P if and only if Q .” Identify P and Q carefully.

It is necessary and sufficient that a matrix A have nonzero determinant for it to be invertible.

11. Make a truth table for the statement $(Q \vee R) \Leftrightarrow (R \wedge Q)$.

12. Negate the following sentence:

There exists a real number a for which $a + x = x$ for all real numbers x .