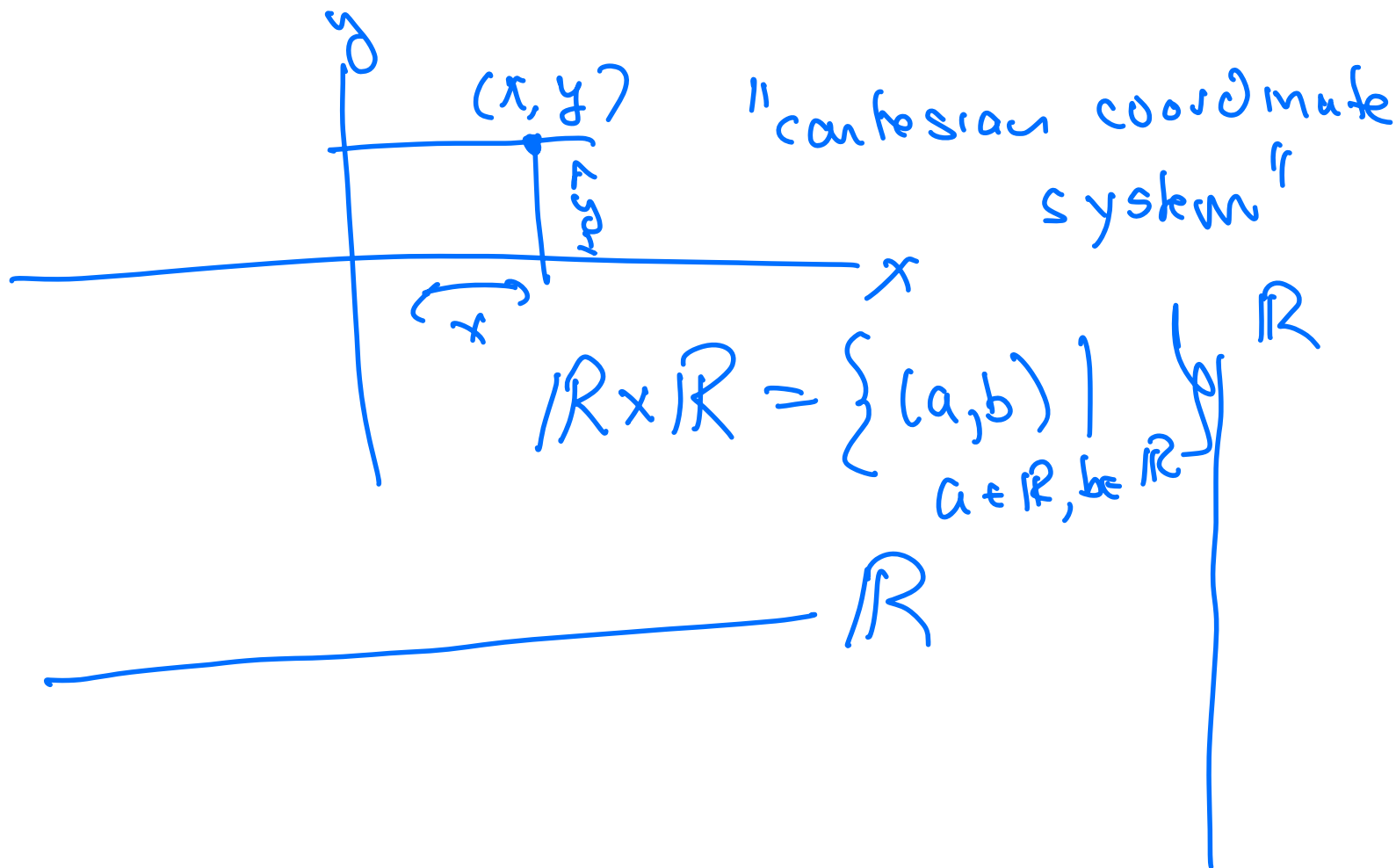


Cartesian Products

Cartesian Products

René Descartes



Definition

Definition: The Cartesian Product $A \times B$ of two sets A and B is the set of ordered pairs (a, b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Example

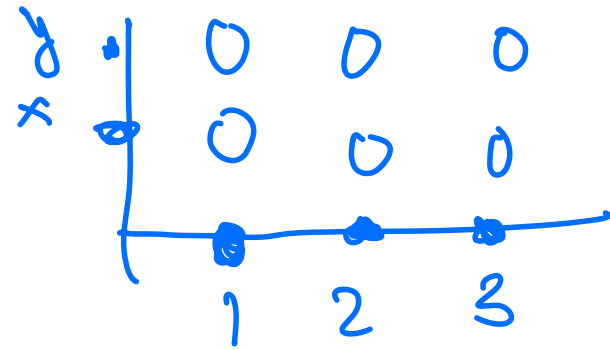
$A = \{1, 2, 3\}$ and $B = \{x, y\}$. What is $A \times B$?

$$A \times B = \{ (u, v) \mid u \in \{1, 2, 3\} \text{ and } v \in \{x, y\} \}$$

$$(1, x) \quad (1, y)$$

$$(2, x) \quad (2, y)$$

$$(3, x) \quad (3, y)$$



Example

$$\mathbb{R} \times \mathbb{R} = \{(a, b) : a \in \mathbb{R}, b \in \mathbb{R}\}$$

$$(3, 7) \neq (7, 3)$$

Example

What is $\mathbb{N} \times \{-1, 1\}$?

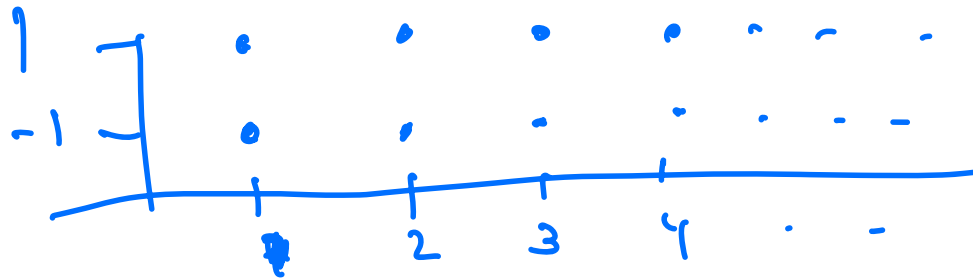
e.g.

$$(1, 1) \in \mathbb{N} \times \{-1, 1\}$$

$$(5, 1)$$

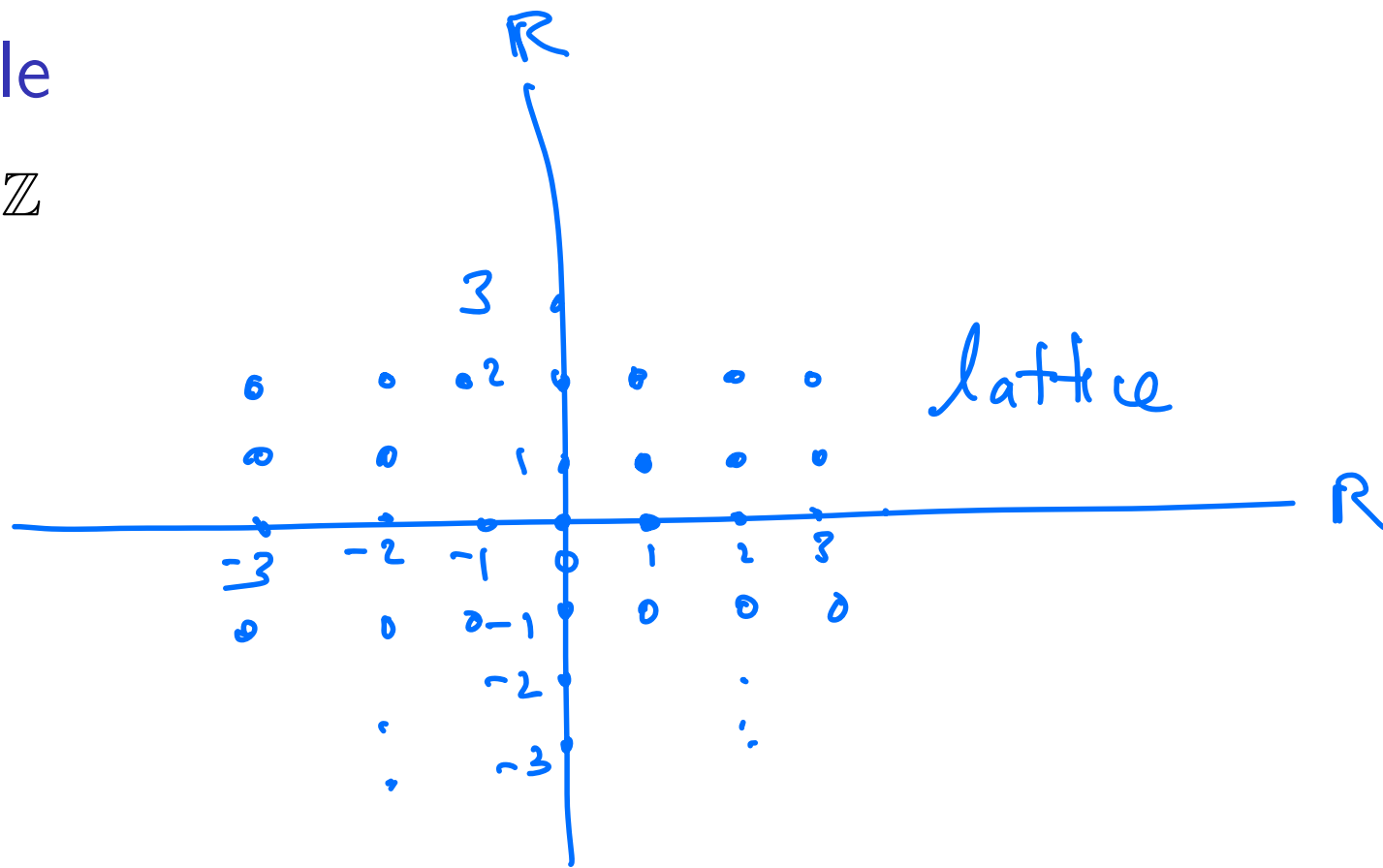
$$(1, 1), (2, 1), (3, 1), \dots$$

$$(1, -1), (2, -1), \dots$$



Example

$$\mathbb{Z} \times \mathbb{Z}$$



Example

$$\overbrace{N \times (N \times N)}^{= N^3} \text{ vs } \underbrace{N \times N \times N}_{= N^3}$$

$$(x, (n, m)) \quad \{(n_1, n_2, n_3) \mid n_1 \in N, n_2 \in N, n_3 \in N\}$$

||?

$$(x', (n', m'))$$

X set

X^n — means

$\underbrace{X \times X \times \dots \times X}_{n \text{ times}}$

$\updownarrow$
n-tuples
 $(x_1, x_2, \dots, x_n)$

$$\mathbb{R}^2 \times (\mathbb{R}^3)$$

$$\mathbb{C} \sim \mathbb{R}^2$$

$$\mathbb{C}^2 \sim \mathbb{R}^4$$

Cartesian Powers

$A = \{H, T\}$. What is A^4 ? What is $|A^4|$?

$$A^4 = \left\{ \begin{array}{l} (H, H, H, H) \\ (H, H, H, T) \\ (H, H, T, H) \\ (H, H, T, T) \\ \vdots \\ (T, T, T, T) \end{array} \right\}$$

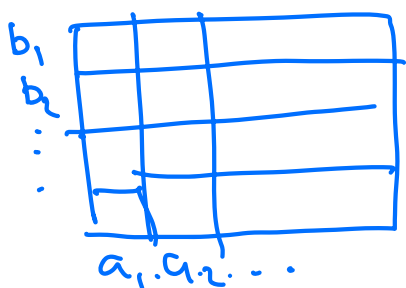
If A and B finite

then

$$|A \times B| = |A| |B|$$

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

$|A|$ choices for a
for each choice you get $|B|$ choices of b .



$$\begin{aligned} |A^n| &= |A \times (A \times \dots \times A)| \\ &= |A| |A \times \dots \times A| \end{aligned}$$