MATH 2710: TRANSITION TO HIGHER MATHEMATICS
SECOND MIDTERM EXAM, NOVEMBER 6, 2019.

Instructions: This exam is worth a total of 100 points. You must do problems 1 through 5, worth a total of 60 points. You may do any two of the three problems 6, 7, or 8, worth 20 points each. I will count your best two scores among these three problems.

Name:

1. (10 points) Let $a(n)$ be a sequence of rational numbers. State precisely what it means to say that $a(n)$ converges to a limit L.

2. (10 points) Prove that, if n is a perfect cube, then $n \equiv \pm 1 \pmod{7}$.

3. (10 points) Find the remainder when 2^{38} is divided by 11.

4. (10 points) Consider the infinite repeating decimal $.\overline{1324}$.

(a) (5 points) Identify the infinite geometric series represented by this decimal (provide a and r).

(b) (5 points) What fraction does this series converge to?

5. (20 points) Give a complete proof that the sequence $a(n) = n/(n-1)$ converges to 1.

6. (20 points) Suppose m is a prime number, and a, b, and x are integers with $x \not\equiv 0 \pmod{m}$.

(a) (10 points) Prove that if $ax \equiv bx \pmod{m}$ then $a \equiv b \pmod{m}$.

(b) (10 points) Give a counterexample to this statement if m is not a prime.

7. (20 points) Let S be a set and let $\sim$ be an equivalence relation on S. Choose x and y in S and let $[x]$ and $[y]$ be the associated equivalence classes, so that, for example,

$$[x] = \{u \in S : u \sim x\}.$$

Being explicit about where you use the properties of an equivalence relation, prove that:

(a) (10 points) if $x \sim y$ then $[x] = [y]$.

(b) (10 points) if $x \not\sim y$ then $[x] \cap [y] = \emptyset$

8. (20 points) Define a function $f : \mathbb{P} \to \mathbb{P}$ recursively by setting $f(1) = 5$ and $f(n) = 2f(n-1) + 1$. Prove that $f(m) = (5)(2^{m-1}) + 2^{m-1} - 1$.