Math 2710: Transitions to Higher Mathematics
First Exam, September 30, 2019

Instructions

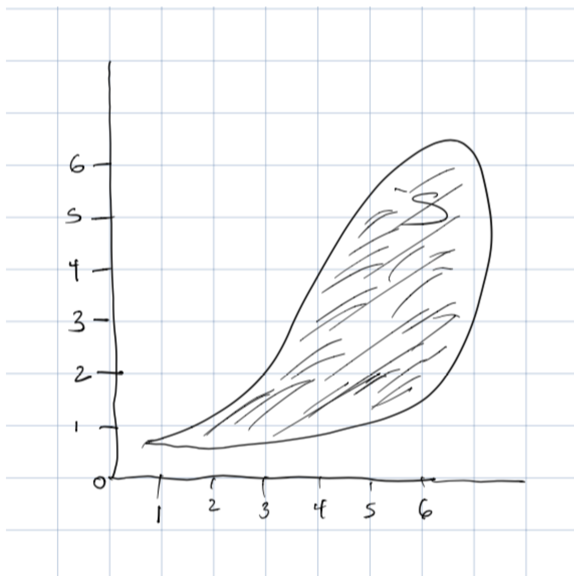
There are a maximum of 100 available points on this exam. You must do all of problems 1-4. You may do any three of the four problems 5-8 and receive full credit. Your score will be based on problems 1-4 together with the *highest three scores* on the four problems 5-8.

Name:

1. (10 points) Let P, Q, and R be propositions. Prove that:

$$(P \implies (Q \text{ OR } R)) \iff ((P \implies Q) \text{ OR } (P \implies R))$$

2. (10 points) Let S be the set of points in $\mathbf{R}^2$ lying in the shaded region in the picture below.



Indicate whether each of the following propositions are true or false:

T **F** For all $2 < x < 3$, there exists $0 \leq y \leq 6$ so that $(x, y) \in S$.

T **F** There exists $0 \leq x \leq 5$ so that for all $0 \leq y \leq 6$, $(x, y) \in S$.

T **F** For all $4 \leq x \leq 5$ and for all $0 \leq y \leq 2$, $(x, y) \in S$.

3. (10 points) Find all solutions (x, y) to the diophantine equation $11x+4y = 1$.

4. (10 points) Write 21 in base 2.

5. (20 points) Prove that, if d, a, and b are integers with $d|a$ and $d|b$, then $d|(ax + by)$ for all integers x and y.

6. (20 points) Let a and b be integers with $b \neq 0$. Let r be the remainder obtained from the division algorithm when dividing a by b.

a. (15 points) Prove that $\gcd(a, b) = \gcd(b, r)$.

b. (5 points) How is this result used in the proof of Euclid's algorithm?

7. (20 points) Let m and n be positive integers, and let p be a prime number. Prove that if $p|\text{lcd}(m,n)$ then $p|m$ or $p|n$.

8. (20 points) Let N be an integer greater than one, and let d be the smallest divisor of N that is greater than one. Prove that d is prime.