1. $\mathbb{Z}$ with multiplication is <u>NOT</u> a group.

- binary operation (multiplication)
- $a(bc) = (ab)c$
- has an identity element 1 $\quad 1\cdot a = a = a\cdot 1$.
- $0 \in \mathbb{Z}$ does not have an inverse.

  $0\cdot x = 1$ has no solution.

  $2 \in \mathbb{Z}$ does not have an inverse. $(\frac{1}{2} \notin \mathbb{Z})$

2. $GL_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; \begin{matrix} a,b,c,d \in \mathbb{R} \\ ad - bc \neq 0 \end{matrix} \right\}$ <u>is</u> a group.

$H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; \begin{matrix} a,b,c,d \in \mathbb{Z} \\ da - bc \neq 0 \end{matrix} \right\}$ is <u>NOT</u> a group.

$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \in H$ but $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\boxed{2x = 1}$ $\quad y = z = 0$

$w = 1$

$\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix} \in GL_2(\mathbb{R})$ but not in H. $\qquad$ has no $\mathbb{Z}$ solutions.

3. Cross product is not associative.

$\vec{v} = (a\hat{i} + b\hat{j} + c\hat{k})$

$\vec{w} = (r\hat{i} + s\hat{j} + t\hat{k})$

$$\vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ a & b & c \\ r & s & t \end{vmatrix}$$

$$= (bt - cs)\hat{i} + (rc - at)\hat{j} + (as - br)\hat{k}.$$

$\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$

$i \times j = k \quad j \times k = i \quad k \times i = j$

$i \times i = j \times j = k \times k = 0.$

$$(\underbrace{(\overset{a}{i} + j) \times \overset{b}{i}}) \times \overset{c}{k} = (-k) \times k = 0$$

$$(i + j) \times (i \times k) = (i + j) \times (-j) = -k$$