Disproof of existence

The proof of Fermat's Last Theorem is a disproof of existence; it shows that there are NO solutions to the Fermat equation.

The disproof of a statement

"There exists $x \in S$ such that $P(x)$"

requires proving a universal statement:

"For all $x \in S$, not $P(x)$."

Disproof of existence

Claim: There exists a pythagorean triple (a, b, c) such that all of a, b, and c are odd.

(a is odd and b is odd and c is odd)

The negation of this claim is

"For all pythagorean triples (a, b, c), at least one of a, b, or c is even."

~(a odd and b odd and c odd)

a even or b even or c is even

Proof: (a, b, c) are a pythagorean triple so $c^2 = a^2 + b^2$.

1. if a is even, then our prop is true.
2. if a is odd and b is even, our prop is still true
3. if a and b are both odd then a^2, b^2 are odd so $c^2 = a^2 + b^2$ is even. If c^2 is even, then c is even. ~~If $c^2 = 2$~~

Disproof of existence by contradiction

Proof by contradiction is often useful to prove "nonexistence" of something.

Claim: There is a real number x such that $x \in (x^4, x^2)$. (See Example 9.5).

$x^4 < x$ and $x < x^2$.

Disproof of claim by contradiction.

so assume $x \in (x^4, x^2)$.

① $x^4 < x$ so x is positive because $x^4 \geq 0$

$x > x^4 \geq 0$ so $x > 0$.

② $x < x^2$ so $\boxed{1 < x}$

③ $x^4 < x$ so $x^3 < 1$

so $(x^3 - 1) < 0$

$(x-1)\underline{(x^2+x+1)} < 0$

$x^2+x+1 > 0$ since $x > 0$

so $x - 1 < 0$

$\boxed{x < 1}$

$x \in (x^4, x^2) \Rightarrow \boxed{x > 1 \text{ and } x < 1}$

so $x \notin (x^4, x^2)$.