Disproof

Disproof

A "disproof" of a statement P is a proof of $\sim P$.

Suppose the result we are interested is a universally quantified statement of the form:

For all $x \in S$, $P(x)$

The negation of this statement is:

There exists $x \in S$ such that $\sim P(x)$.

Disproof

For example, if the original statement is:

- if $n \in \mathbb{Z}$ and $n^5 - n$ is even, then n is even.

For all $n \in \mathbb{Z}$,

if $n^5 - n$ even, then n is even Q

P

The negation is:

- There exists an integer n, such that $n^5 - n$ is even and n is odd.

P $\sim Q$

There exists $n \in \mathbb{Z}$, such that $n^5 - n$ is even and n is odd

$\sim (P \Rightarrow Q)$

$\Updownarrow$

P and $\sim Q$

Disproof by counterexample

The negation of the "for all statement" is a "there exists" statement. To prove that negation, we need to *find an example that satisfies the negation.*

To disprove

- if $n \in \mathbb{Z}$ and $n^5 - n$ is even, then n is even. FALSE. $n=1$ is a counterexample.

we must find an integer n such that $n^5 - n$ is even and n is odd.

Try a few n and it doesn't take long to find $n = 1$.

Let $n = 1$. Then $n^5 - n = 0$ is even, but $n = 1$ is odd.

This example which establishes the truth of the negation is called a *counterexample* to the original statement.

Another disproof by counterexample

It may not be obvious that a statement is false. (this is problem 7 on page 179).

Proposition: Suppose that A, B, and C are sets. If $A \times C = B \times C$ then $A = B$.

$A \times C = \{(a,c) : a \in A, c \in C\}$

$B \times C = \{(b,c) : b \in B, c \in C\}$.

Show $A = B$.

① ~~Show~~ $A \subseteq B$. Choose $a \in A$. Pick $c \in C$, so that $(a,c) \in A \times C$. But $A \times C = B \times C$ so $(a,c) \in B \times C$, so $a \in B$.

② $B \subseteq A$ Choose $b \in B$. Pick $c \in C$ so that $(b,c) \in B \times C$. $B \times C = A \times C$, $(b,c) \in A \times C$, therefore $b \in A$.

uh-oh!!

so $A = B$.

Choose $C = \emptyset$. $A \times C = B \times C = \emptyset$.

$A = \{1\}$ $B = \mathbb{R}$ $\{1\} \times \emptyset = \mathbb{R} \times \emptyset = \emptyset$ but $\{1\} \neq \mathbb{R}$.

Counterexamples, cont'd

Counterexamples often come from "edge cases." - What if a variable is zero? - What if a set is empty? - What if an integer is negative?