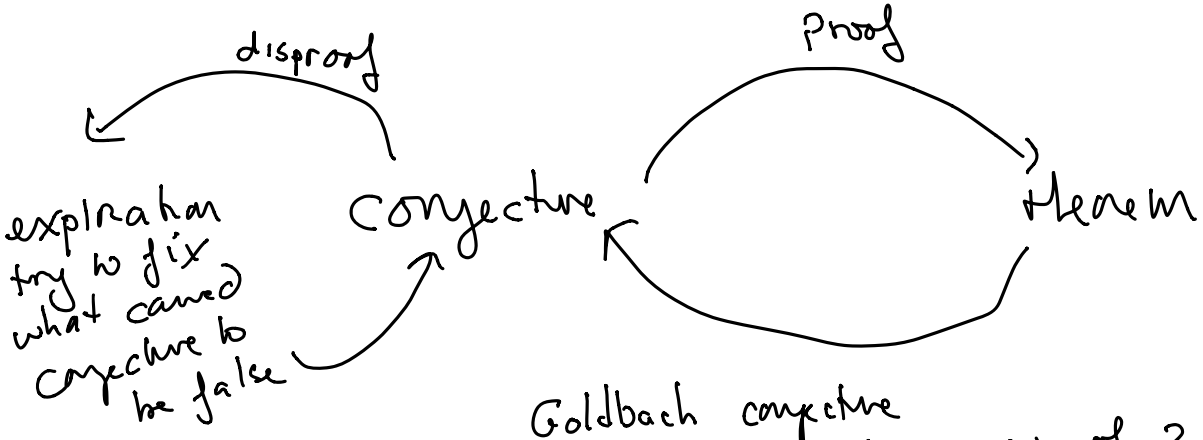
Disproof

The process of mathematics

Mathematics as a science advances through the formulation of conjectures and their proofs and disproof.



Goldbach conjecture
Every even # is a sum of 2 primes

Twin primes
3,5 are twins, 11,13 twins
a twin prime is a pair of primes that are 2 apart. Are there ∞ many pairs

A brief history of a famous problem

- Diophantus publishes *Arithmetica* in Greek Alexandria in the 3rd century CE. (10/13 chapters survive today).
- Work preserved by Arab and Persian scholars for 1000 years.

Book contains discussion of Pythagorean Triples: natural numbers a, b, c such that

$$c^2 = a^2 + b^2.$$

$5^2 = 3^2 + 4^2$

Let $a = u^2 - v^2$, $b = 2uv$ and $c = u^2 + v^2$ where u and v are natural numbers with no common factors. For example $u = 3, v = 2$ so $a = 5$, $b = 12$, and $c = 13$. Then

$u^2 = 9$

$v^2 = 4$

$$c^2 = a^2 + b^2.$$

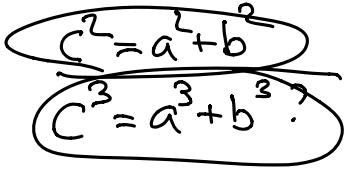
$13^2 = 12^2 + 5^2$

$169 = 144 + 25$

More history

- 6 chapters of Diophantus work translated to Latin in the 16th century.
- Fermat, a french lawyer in Toulouse, becomes interested in mathematics. He makes many contributions. Among them, he reads Diophantus and asks about triples (a, b, c) of natural numbers that satisfy pythagoras-like theorems with higher powers:

$$c^3 = a^3 + b^3$$



or, more generally,

$$c^n = a^n + b^n$$

where $n \geq 3$. He shows that *there are no solutions in natural numbers to this equation with $n = 3$ or $n = 4$.

More history

In the margin of his copy of *Arithmetica* he writes:

"There are no solutions in natural numbers to the equation $c^n = a^n + b^n$ for any $n \geq 3$. I have a wonderful proof of this proposition but the margin of this book is too small to contain it."

This note is found by his son after his death.

If a, b, c are integers, none of them zero, and $n \geq 3$, then

$$c^n \neq a^n + b^n.$$

More history

For 350 years mathematicians tried to find Fermat's proof. This became known as Fermat's Last Theorem, though it should have been called "Fermat's Conjecture."

In the 19th century, a number of mathematicians made progress (Kummer most notably) but the problem remained unsolved.

There were a number of other claimed proofs that turned out to be wrong.

More history

In the 1970's and 1980's, several mathematicians had insight into a new possible way to prove Fermat's theorem.

In 1994, drawing on *all* of the work done before, Andrew Wiles claimed to have found a proof. But it turned out his proof had a mistake in it, too. However, with the help of Richard Taylor, he was able to fix the proof and Fermat's Conjecture *finally* became a theorem after 350 years.

Today

Mathematicians continue to work on many unsolved problems, including the Millenium problems and many, many others.