Set Proofs

Elements of sets

Many types of theorems can be expressed as questions about the relationship between sets. Sometimes it's a question of membership.

Theorem: For any natural numbers a and b there exist integers k and l such that

$$\gcd(a, b) = ak + bl.$$

Theorem: Let a and b be natural numbers, and let $\underline{A} = \{\underline{ax + by : x, y \in \mathbb{Z}}\}$. Then $\underline{\gcd(a, b)} \in \underline{A}$.

$\gcd(a,b) = ax+by$ for some $x, y \in \mathbb{Z}$.

More examples

General situation: $\underbrace{A = \{x \in S : P(x) \text{ is true}\}}$. Then

$$x \in A \Leftrightarrow (x \in S) \wedge P(x).$$

- Let $A = \underbrace{\{3x + 2 : x \in \mathbb{Z}\}}$. Then $14 \in A$.

$3(-3)+2 = -7,$
$3(-2)+2 = -4,$
$3(-1)+2 = -1$
2
$5,$
$\vdots$

is $14 = 3x+2$ for some $x \in \mathbb{Z}$?
$12 = 3x$
$4 = x$

yes because $14 = 3(4)+2$.

$13 \notin A$. Because $13 = 3x+2$, $x = 11/3 \notin \mathbb{Z}$.

- Let $A = \{3x + 2 : x \in \mathbb{Z}\}$. If $x \equiv 2 \pmod 3$, then $x \in A$.

Recall: $x \equiv 2 \bmod 3$ means $x-2$ is divisible by 3.

$x-2 = 3y$ for some integer y.

$x = 3y+2$ for some integer y

so $x \in A$

More examples

- Let B be the set of $X \in \mathcal{P}(\mathbb{N})$ such that, for all $x \in X$ and $y \in X$, $|x - y| < 2$.

Is $\{-1, 2\} \in B$? Is ~~{...}?~~ $\{2,3\}$

$B = \{ \underline{X \in \mathcal{P}(\mathbb{N})} : \text{for all } x, y \in X, \ |x-y| < 2 \}$,

$\{-1, 2\} \in B$?

$\mathcal{P}(\mathbb{N}) = \{ \text{subsets of } \mathbb{N} \}$.

is $\{-1, 2\} \in \mathcal{P}(\mathbb{N})$? $\underline{\underline{\text{NO}}}$ $-1 \notin \mathbb{N}$.

Is $\{2,3\}$ in the set B?

$\{2,3\} \in \mathcal{P}(\mathbb{N})$

x	y	$\|x-y\|$	
2	2	0	< 2
2	3	1	< 2
3	2	1	< 2
3	3	0	< 2

$\{1,2,3,4\} \in B$?

$\{1,2,3,4\} \subseteq \mathbb{N}$ so it is an element of $\wp(\mathbb{N})$.

$|4-1| = 3 > 2$ so $\{1,2,3,4\}$ does not have the property that $|x-y| < 2$ for all $x,y \in X$.