Euclid's Proof

Recall that we know that every integer can be written as a product of prime numbers. In particular, every integer greater than 1 has a prime divisor.

Proposition: There are infinitely many primes.

Euclid's Proof cont'd

Proof: Suppose that there are only finitely many prime numbers.

$\{p_1, \dots, p_{1,000,000,000}\}$

- Multiply them together and let P be their product.
- Consider the integer $P + 1$.
- This integer must have a prime divisor p, which must be greater than one, so we can write $P + 1 = pA$. $A \in \mathbb{Z}$ $\boxed{p > 1}$
- Since P is the product of all the primes, we know that p is a divisor of P, so we can write $P = pB$.
- Therefore $1 = pA - P = pA - pB = p(A - B)$. $\boxed{p = 1}$
- This implies that p is a divisor of 1, so $p = 1$.
- We've proved that $p > 1$ and $p = 1$, which is a contradiction. Thus there cannot be finitely many primes. so there are ∞ many primes.