Checking cases

Cases

Proposition: If n is a natural number, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.

$n = 3 \quad 1 + (-1)^3(5) = 1 - 5 = -4$ ✓

$n = 4 \quad 1 + (-1)^4(7) = 1 + 7 = 8$ ✓

n — n even: $1 + (2n-1) = 2n$ is a multiple of 4

n — n odd: $1 - (2n-1) = 2 - 2n$ is a multiple of 4

If n is even then $2n$ is a multiple of 4 ✓

If n is odd then $2 - 2n$ is a multiple of 4

n even
$n = 2m$
$2n = 4m$ ✓

n odd
$n = 2m+1$
$2 - 2n =$
$2 - 2(2m+1)$
$= 2 - 4m - 2$
$= -4m$ ✓

Cases

Proof: Let $K(n) = 1 + (-1)^n(2n-1)$. We consider the cases where n is odd and even separately. When n is even, $K(n) = 1 + (2n-1) = 2n$. Since n is even, $n = 2m$ for some m, and therefore $K(n) = 2(2m) = 4m$. Therefore $K(n)$ is a multiple of 4.

When n is odd, $K(n) = 1 - (2n-1) = 2 - 2n$. Since n is odd, $n = 2m+1$ for some m, and therefore

$$K(n) = 2 - 2(2m+1) = 2 - 4m - 2 = 4m$$

and again $K(n)$ is a multiple of 4.

Cases

Proposition: (The Triangle Inequality) For any real numbers x and y, we have

$$|x+y| \leq |x|+|y|$$

Note: $|x| = x$ if $x \geq 0$, otherwise $|x| = -x$.

$|-3| = -(-3) = 3$

4 cases
$$\begin{cases} x \geq 0,\ y \geq 0 & |x| = x,\ |y| = y \quad (1) \\ x \geq 0,\ y < 0 \to & |x| = x,\ |y| = -y \quad (2) \\ x < 0,\ y \geq 0 \to & |x| = -x,\ |y| = y \quad (3) \\ x < 0,\ y < 0 & |x| = -x,\ |y| = -y \quad (4) \end{cases}$$

In case 1, $|x| = x$, $|y| = y$, $x+y \geq 0$ so $|x+y| = x+y$
$|x| + |y| = |x+y|$

case 4: $|x| = -x$, $|y| = -y$, $x+y < 0$ $|x+y| = -(x+y)$.
$-(x+y) = -x + (-y)$

In cases 1, 4 $|x+y| = |x| + |y|$

Cases

Proof: There are four cases to consider, depending on the signs of x and y, and we take them in turn.

1. $x \geq 0$ and $y \geq 0$. Then $x + y \geq 0$. Therefore, in this case, $|x| = x$, $|y| = y$, and $|x + y| = x + y$ and so $|x + y| = |x| + |y|$.
2. If $x < 0$ and $y < 0$ then $|x + y| = -x - y = |x| + |y|$.

Cases

3. $x \geq 0$ and $y < 0$.

non-negative

$|x+y|$ ~~positive~~ or negative?

$x = 15 \quad y = -5 \quad |x+y| = |10| = 10 \leq |x|+|y|$

$= 15+5 = 20$

$x = 15 \quad y = -20 \quad |x+y| = |-5| = 5$

$|x|+|y| = 15+20 = 35$

$x+y \geq 0 \quad |x+y| = x+y = |x|-|y| \leq |x|+|y|$

$x+y < 0 \quad |x+y| = -(x+y) = -x-y = -|x|+|y| \leq |y|+|x|$

Cases

3. $x \geq 0$ and $y < 0$.

~~Then $y \leq x + y < x$.~~ If $x + y \geq 0$, then $|x + y| = x + y = |x| - |y| \leq |x| + |y|$. If $x + y < 0$, then $|x + y| = -x - y = -|x| + |y| \leq |x| + |y|$.

The 4th case, $x < 0$ and $y \geq 0$, follows by the same argument.

There are 2 cases remaining, where x and y have opposite signs. Without loss of generality, suppose $x \geq 0$ and $y < 0$. . . . . .