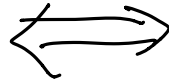


Definitions

An example from linear algebra

Definition: A set $\{v_1, \dots, v_n\}$ of elements of a vector space V is called *linearly independent* if, for any set $\{a_1, \dots, a_n\}$ of scalars, if $\sum_{i=1}^n a_i v_i = 0$ then $a_i = 0$ for all $i = 1, \dots, n$. A set of vectors that is not linearly independent is called *linearly dependent*.

A set $\{v_1, \dots, v_n\}$
is linearly independent
P



For any set $\{a_1, \dots, a_n\}$ of
scalars such that
 $\sum_{i=1}^n a_i v_i = 0,$
we have $a_i = 0, i = 1, \dots, n.$

If $\sum a_i v_i = 0$ then all $a_i = 0,$
Q R

$[P \Leftrightarrow (Q \Rightarrow R)]_{\text{true}}$

Linearly Independent
NOT " "

Q \Rightarrow R

False that Q \Rightarrow R

Theorem: The set of vectors $\{(1, 3), (2, 2)\}$ is linearly independent in \mathbb{R}^2 .

Proof: We must show that

$$a_1(1, 3) + a_2(2, 2) = \vec{0} \implies a_1 = 0 \text{ and } a_2 = 0$$

($\sum a_i v_i$) Assume $a_1(1, 3) + a_2(2, 2) = \vec{0}$

$$\text{Therefore } (a_1, 3a_1) + (2a_2, 2a_2) = \vec{0}$$

$$(a_1 + 2a_2, 3a_1 + 2a_2) = \vec{0}$$

$$a_1 + 2a_2 = 0$$

$$3a_1 + 2a_2 = 0.$$

$$2a_1 = 0 \quad \text{so } a_1 = 0$$

$$\text{Then } 2a_2 = 0 \quad \text{so } a_2 = 0$$

$$\text{so } a_1 = a_2 = 0.$$

Therefore the set is linearly independent.

Theorem: The set of vectors $\{(1, 1, 1), (2, 2, 2), (1, 3, 2)\}$ is linearly dependent in \mathbb{R}^3 .

Proof: We must show that

$$\sum a_i v_i = 0 \Rightarrow \text{all } a_i = 0 \quad \text{is FALSE}$$

P Q

P	Q	$P \Rightarrow Q$
T	F	F
:		T

$P \Rightarrow Q$ is FALSE if P is True and only if Q is FALSE

$$\sum a_i v = 0 \quad \text{and not all } a_i = 0.$$

$$a_1(1, 1, 1) + a_2(2, 2, 2) + a_3(1, 3, 2) = 0.$$

$$(a_1 + 2a_2 + a_3, a_1 + 2a_2 + 3a_3, a_1 + 2a_2 + 2a_3) = 0.$$

$$a_1 = 2 \quad a_2 = -1 \quad a_3 = 0.$$

$$\text{Then } a_1(1, 1, 1) + a_2(2, 2, 2) + a_3(1, 3, 2) = 0.$$

We've shown that the vectors are linearly dependent.