Direct Proofs

If P, then Q

Proposition: If P is true, then Q is true.

We are asserting the truth of the implication $P \implies Q$. If P is false, then the implication is automatically true. So from a practical point of view, we must show that whenever P is true, Q is also true.

Proof: Suppose P.

(bunch of stuff)

Therefore Q.

$P \Rightarrow P_1 \Rightarrow P_2 \Rightarrow P_3 \cdots \Rightarrow P_n \Rightarrow Q$

Simple example

Proposition: The sum of two odd integers is even.

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$\forall$ integers a, b such that a, b are odd, then $a+b$ is even.

Proof:

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{P}$, then $\underbrace{a+b \text{ is even}}_{Q}$.

Suppose a and b are odd integers.

Therefore $a + b$ is even.

Proposition: The sum of two odd integers is even.

Proof:

Definition: a is odd if there is an integer k so that $a=2k+1$.

Suppose a and b are odd integers.

Then there are integers k and u so that $a = 2k + 1$ and $b = 2u + 1$.

Therefore $a + b$ is even.

Proposition: The sum of two odd integers is even.

Proof:

Suppose a and b are odd integers.

Then there are integers k and u so that $a = 2k + 1$ and $b = 2u + 1$.

Therefore

$$a + b = (2k + 1) + (2u + 1) = 2(k + u) + 2 = 2(k + u + 1)$$

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Therefore $a + b = (2k + 1) + (2u + 1) = 2(k + u) + 2 = 2(k + u + 1)$

The definition of even integer says that an integer x is even if there is an integer y so that $x = 2y$. We found that $a + b = 2(k + u + 1)$ where $k + u + 1$ is an integer.

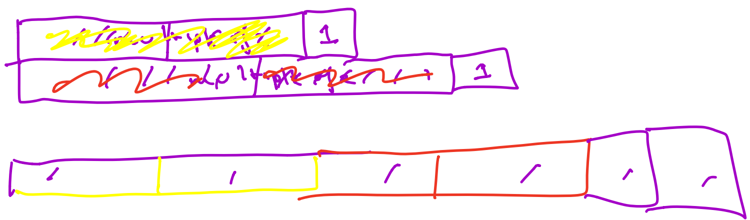
Definition: a is odd if there exists $k \in \mathbb{Z}$ so that $a = 2k+1$

Definition: a is even if there exists $k \in \mathbb{Z}$ so that $a = 2k$.

Therefore $a + b$ is even.

a, b odd $\rightsquigarrow$ a+b even.

3	7	3+7 = 10	10 even ✓
5	11	5+11 = 16	16 even ✓



algebra

I get it!