

Direct Proof

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Basic terminology

- ▶ Definitions, Theorems, Proofs
- ▶ Axioms
- ▶ Propositions, Lemmas, Corollaries
- ▶ Conjectures

Theorems

- ▶ Theorem. A theorem is a mathematical statement that is proven to be true.

Theorem: The sum of the squares of the lengths of the sides of a ~~right triangle~~ is equal to the square of the length of the hypotenuse.

Theorem: The sum of two even integer is an even integer. }

If a, b are integers, and a, b are even then $a+b$ is even.

Definitions

- ▶ Definition. A definition is “an exact unambiguous explanation of the meaning of a mathematical word or phrase.”

Definition: A “right triangle” is a triangle one of whose interior angles is a right angle. *If a triangle has an interior angle that is a right angle then it is called a “right triangle.”*

Definition: A positive integer n is prime if it is greater than one and its only divisors are 1 and n . *If n is an integer greater than one, and its only divisors are one and n , then*

Definition: A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$. *n is prime.*

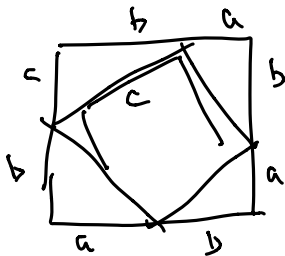
- ▶ Definitions are sometimes written as “If, Then” but they are really “If and only if” statements.

Proofs

A proof is a logical argument that establishes the truth of a theorem.

A true proof of a mathematical statement is almost never given because of length. In practice a proof describes the key steps that are needed to construct a formal proof. There is a social element in what constitutes a proof which depends on the audience.

Recently some mathematicians have been advocating for computer verified proofs because mistakes do occur in published results.



4 right triangles

$$c^2 + c^2 = (a+b)^2$$

$$a^2 + b^2 = c^2$$

$$4 \cdot \frac{1}{2} (ab) + c^2 = a^2 + 2ab + b^2$$

$$c^2 = a^2 + b^2$$

Lemma, Proposition, Corollary

- ▶ Lemmas and Propositions are words for “less important” theorems. “Lemma” usually refers to a small theorem that is needed to prove a bigger one. “Proposition” is bigger than “lemma” but smaller than “Theorem.”
- ▶ Corollary is a word for a theorem that is an immediate consequence of a Theorem.

Lemmas and Propositions precede theorems; corollaries follow them.

Theorem: Any polynomial function is continuous.

Corollary: Any quadratic function is continuous.

Conjecture "Eduated Guess"

Goldbach's Conjecture

Every even number > 2 is a sum of
2 primes.