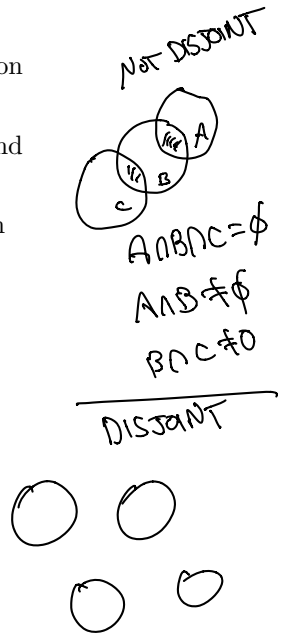
Addition/Subtraction Principle

Addition Principle and Disjoint Unions

Definition: Two sets are said to be *disjoint* if $X \cap Y = \emptyset$. Similarly, a collection $X_1, \ldots, X_n$ of sets are said to be disjoint if any pair of them is disjoint.

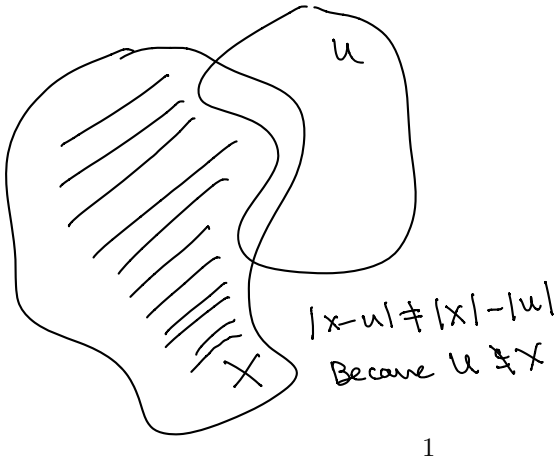


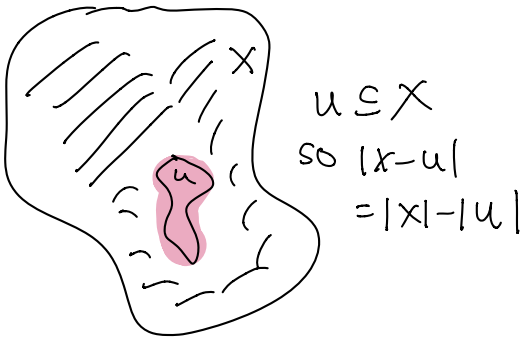
Note that one can have 3 sets A, B, and C with $A \cap B \cap C = \emptyset$ but A, B, and C are not disjoint.

Proposition: Let $X_1, X_2, \ldots, X_n$ be a disjoint collection of finite sets. Then

$$|X_1 \cup X_2 \cup \cdots \cup X_n| = \sum_{i=1}^{n} |X_i|.$$

Proposition: Suppose that $U \subseteq X$. Then $|X - U| = |X| - |U|$





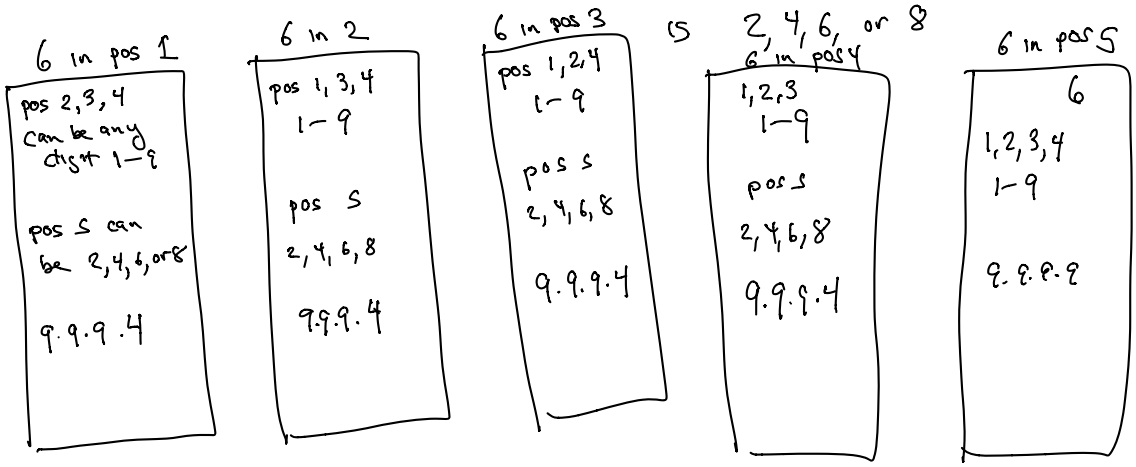
Examples

(Example 3.6) How many even 5 digit numbers are there for which:

- no digit is zero
- the digit 6 appears exactly once.

we've only using 1,2,3,4,5,6,7,8,9

even means last digit



$4(9 \cdot 9 \cdot 9 \cdot 4) + 9 \cdot 9 \cdot 9 \cdot 9$

Problem 3

(Problem 3 from section 3.3) Five cards are dealt from a 52-card deck and lined up in a row. How many such lineups are there in which all five cards are the same color (i.e. black or red)?

26 red
26 black

Total Number =

where all are red : 26 in position 1 — $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21$
25
24
23
22

+

where all are black — same for all black

total $= 2\left(26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21\right)$

Problem 9

(Problem 9 from Section 3.3) Consider "words" of length 6 made from the letters A, B, C, D, E, F, G, H. How many such words are possible if each letter can occur at most one time, and the word must contain two consecutive vowels?

WORD CONTAINS AE
OR
CONTAINS EA

How many words of length 6 contain AE and the letters chosen from B, C, D, F, G, H, each only occurring once.

A E _ _ _ _ 6 choices, 5, 4, 3 — group has 6·5·4·3 elements.

A E
A E
A E
A E

6·5·4·3
6·5·4·3
6·5·4·3
6·5·4·3

5(6·5·4·3)

Same thing with EA : same count.

Final #: 2 · 5 · (6·5·4·3)