

Multiplication Principle

Problem 5

Consider 8 digit binary strings such as 10011011 or 00001010.

- How many such strings are there?

$$|\overbrace{Y \times Y \times Y \dots \times Y}^8| = 2^8$$

$$|Y_1 \times Y_2 \times \dots \times Y_n| = |Y_1| \dots |Y_n|$$

$$Y = \{0, 1\}$$

look at elements of

$$\underbrace{Y \times Y \times \dots \times Y}_{8 \text{ times}}$$

$$Y = \{0,1\}$$

- How many such strings end in 0

such a string is an element of

$$|Y \times Y \times Y \times Y \times Y \times Y \times \{0\}| = 2^7$$

2 2 2 2 2 2 1

- How many such strings have 1's for their second and fourth digits

$$Y = \{0, 1\}$$

$$\left| \underset{2}{Y} \times \underset{1}{\{1\}} \times \underset{2}{Y} \times \underset{1}{\{1\}} \times \underset{2}{Y} \times \underset{2}{Y} \times \underset{2}{Y} \times \underset{2}{Y} \right| = 2^6$$

- How many such strings have 1's for their second OR fourth digits.

	2 nd digit	4 th digit	
$\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}$	1	0	2^6
$\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}$	0	1	2^6
$\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}$		1	2^6
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			$3 \cdot 2^6$

Count strings with a 0 in both the 2nd and 4th positions.

$$\begin{aligned}
 \frac{\text{all strings}}{2^8} &= \frac{\begin{array}{|l} 0 \text{ in } 2^{\text{nd}} \\ \text{and} \\ 0 \text{ in } 4^{\text{th}} \end{array}}{2^6} + \frac{\begin{array}{|l} 1 \text{ in either } 2^{\text{nd}} \\ \text{or } 4^{\text{th}} \end{array}}{2^8 - 2^6} \\
 &= 2^6 (2^2 - 1) \\
 &= \underline{\underline{2^6 \cdot 3}}
 \end{aligned}$$