Multiplication Principle

Problem 5

Consider 8 digit binary strings such as 10011011 or 00001010.

- How many such strings are there?

$Y = \{0, 1\}$

look at Elements of

$$\underbrace{Y \times Y \times \cdots \times Y}_{8 \text{ times}}$$

$$\left| \overbrace{Y \times Y \times Y \cdots \times Y}^{8} \right| = 2^8$$

$$\left| Y_1 \times Y_2 \times \cdots \times Y_n \right| = |Y_1| \cdots |Y_n|$$

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$$Y = \{0, 1\}$$

- How many such strings end in 0

such a string is an element of

$$\left| Y \times Y \times Y \times Y \times Y \times Y \times \{0\} \right| = 2^7$$

$$\;\;\;\; 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad\quad 1$$

- How many such strings have 1's for their second and fourth digits

$$Y = \{0, 1\}$$

$$\left| \underset{2}{Y} \times \underset{1}{\{1\}} \times \underset{2}{Y} \times \underset{1}{\{1\}} \times \underset{2}{Y} \times \underset{2}{Y} \times \underset{2}{Y} \times \underset{2}{Y} \right| = 2^6$$

- How many such strings have 1's for their second OR fourth digits.

$$2^{nd}\ digit \qquad 4^{th}\ digit$$

$Y \times \{1\} \times Y \times \{0\} \times Y \times Y \times Y \times Y$. $\quad 1 \qquad\qquad 0 \qquad\qquad 2^6$

$Y \times \{0\} \times Y \times \{1\} \times Y \times Y \times Y \times Y$ $\quad 0 \qquad\qquad 1 \qquad\qquad 2^6$

$Y \times \{1\} \times Y \times \{1\} \times Y \times Y \times Y \times Y$ $\quad 1 \qquad\qquad 1 \qquad\qquad \dfrac{2^6}{3 \cdot 2^6}$

Count strings with a $\underline{0}$ in both the 2nd and 4th position.

$$\boxed{\begin{array}{c} all \\ strings \end{array}} \atop 2^8 \quad = \quad \boxed{\begin{array}{c} 0\ in\ 2^{nd} \\ and \\ 0\ in\ 4^{th} \end{array}} \atop 2^6 \quad + \quad \boxed{\begin{array}{c} 1\ in\ either\ 2^{nd} \\ or\ 4^{th} \end{array}}$$

$$2^8 - 2^6 = 2^6(2^2 - 1)$$
$$= \underline{2^6 \cdot 3}$$

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