Translating

Example 2.8

Mean Value Theorem

Theorem: [If $f : \mathbb{R} \to \mathbb{R}$ is continuous on the interval $[a, b]$ and differentiable on (a, b), then there is a number $c \in (a, b)$ for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$]

[$f : \mathbb{R} \to \mathbb{R}$
f is continuous on $[a,b]$ $\mathcal{C}(f)$
f is differentiable on (a,b) $\mathcal{D}(f)$] $\Longrightarrow$ [there exists $c \in (a,b)$ so that $f'(c) = \frac{f(b)-f(a)}{b-a}$]

$$\left[\forall f : \mathbb{R} \to \mathbb{R},\ \left(\mathcal{C}(f) \text{ and } \mathcal{D}(f)\right) \Longrightarrow \left(\exists c,\ c \in (a,b),\ f'(c) = \frac{f(b)-f(c)}{b-a}\right)\right]$$

Example 2.9

Conjecture: Every even integer greater than 2 is the sum of two primes.

GOLDBACH'S CONJECTURE

- $4 = 2 + 2$

$6 = 3 + 3$

$8 = 5 + 3$

$12 = 7 + 5$

$\vdots$

For all $x \in \mathbb{Z}$, and x even, and $x > 2$

- x even
- $x > 2$

$P \subseteq \mathbb{Z}$

$P = \{x : x \in \mathbb{Z}, x \text{ prime}\}$

$\exists p \in P, \exists q \in P, x = p + q$

$\forall x \in \mathbb{Z} \left[(x \text{ even}) \text{ and } (x > 2) \Longrightarrow \exists p, q \in P, x = p + q \right]$

Problem 2.3

If x is prime then $\sqrt{x}$ is not rational.

Textbook answer: $P \implies \sim Q$ where $P(x)$ is "x is prime" and $Q(x)$ is "$\sqrt{x}$ is a rational number."

$P(x)$: x is a prime number

$Q(x)$: $\sqrt{x}$ is a rational number

Alternative:

$$[\forall x, x \text{ prime}, P(x) \Rightarrow \sim Q(x).]$$

$Q(x)$ "$\sqrt{x}$ is a rational number".

$$\sqrt{x} \in \mathbb{Q} \qquad (\sqrt{x})^2 = x$$

$\sqrt{x}$ rational : $\exists y \in \mathbb{Q}$ such that $y^2 = x$.

$\forall x \in \mathbb{Z}$, $(x \text{ is prime}) \Rightarrow \sim(\exists y \in \mathbb{Q}$ such that $y^2 = x)$

$\forall x \in \mathbb{Z}$ $(x \text{ is prime}) \Rightarrow \forall y \in \mathbb{Q}, y^2 \neq x$.

Problem 2.13

"Everything is funny as long as it is happening to someone else."

Textbook answer :

$$\forall x, (\sim M(x) \wedge S(x)) \implies F(x)$$

where $M(x)$ means "x is happening to me", $S(x)$ is "x is happening to someone", $F(x)$ means "x is funny."

Alternative: