Implications and open sentences

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Consider the statement:

$P(x)$ $Q(x)$

For all $x \in \mathbb{Z}$, if x is divisible by 6 then x is even.

- "x is divisible by 6" is an open sentence $P(x)$ ✓
- "x is even" is an open sentence $Q(x)$ ✓
- "if x is divisible by 6 then x is even" is an open sentence $P(x) \implies Q(x)$.

Analysis of quantified implication

"For all $x \in \mathbb{Z}$, if x is divisible by 6 then x is even"

is a statement that "ands" together $\underline{P(x) \implies Q(x)}$ as x runs over the integers:

P	Q	P ⟹ Q
T	T	T
T	F	F
F	T	T
F	F	T

$$\cdots (P(\underbrace{-5) \Rightarrow Q(-5}_{}))\wedge(P(-4) \Rightarrow Q(-4))\wedge\cdots\wedge(P(3) \Rightarrow Q(3))\cdots$$

−5 div by 6 ⇒ −5 even
TRUE

−4 div by 6 ⇒ −4 even

$\underline{P(6)} \Rightarrow \underline{Q(6)}$

This will be true if every one is true, meaning one of the following is true:

- x is not divisible by 6, so $P(x)$ is false for that x
- x is divisible by 6 and x is even meaning $P(x)$ and $Q(x)$ are true for that x.

It will be false if there is at least one x that is divisible by 6 but not even.

Conventional interpretation

It is a common convention to read a statement like

"If x is an integer divisible by 6, then x is even" — For all $x \in \mathbb{Z}$, if x is divisible by 6 then x is even.

as including an implicit quantifier "for all $x \in \mathbb{Z}$."

- If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function, then it is continuous.
- If $x \in \mathbb{R}$, then $x^2 = x$ implies $x = 0$ or $x = 1$.

For all functions $f : \mathbb{R} \to \mathbb{R}$, if f is differentiable then f is continuous.

If $x \in \mathbb{R}$, then $x^2 = x$ implies $x = 0$ or $x = 1$.

$\forall x \in \mathbb{R}, [x^2 = x \Rightarrow x = 0 \text{ OR } x = 1.]$