Pairs of Quantifiers

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Paired quantifiers $\exists, \exists$

- There exists $x \in A$ so that there exists $y \in B$ so that $P(x, y)$

There exists $x \in \mathbb{N}$ so that there exists $y \in \mathbb{N}$ so that $x + y = 5$.

$x = 2$ $y = 3$ shows statement is TRUE

$\exists (x, y) \in \mathbb{N} \times \mathbb{N}, \quad x + y = 5$

$\forall, \forall$

- For all $x \in A$ and for all $x \in B$, $P(x, y)$.

For all $x \in \mathbb{N}$ and for all $y \in \mathbb{N}$, $xy > 0$.

For all $x \in \mathbb{Z}$ and for all $y \in \mathbb{N}$, $xy > 0$.

For all $x \in \mathbb{N}$ and for all $y \in \mathbb{N}$, $xy > 0$ TRUE
because product of positive numbers is positive

For all $x \in \mathbb{Z}$
For all $y \in \mathbb{N}$
$xy > 0$ FALSE
$x = -1$, $y = 1$ $\quad -1 \cdot 1 \not> 0$

For all $(x, y) \in \mathbb{Z} \times \mathbb{N}$, $xy > 0$ – FALSE

$\forall, \exists$

- For all $x \in A$ there exists $y \in B$ so that $P(x, y)$.

For all $x \in \mathbb{N}$ there exists $y \in \mathbb{N}$ so that $2y = x$.

For all $x \in \mathbb{Z}$ there exists $y \in \mathbb{Q}$ so that $2y = x$.

For all $\epsilon \in \mathbb{R}$ with $\epsilon > 0$, there exists $\delta \in \mathbb{R}$ with $\delta > 0$ so that $x^2 < \epsilon$ when $x < \delta$.

For all $x \in \mathbb{N}$, [there exists $y \in \mathbb{N}$ so that $2y = x$.]

FALSE : $x = 5$

there exists $y \in \mathbb{N}$ so that $2y = 5$ is FALSE

For all $x \in \mathbb{Z}$ [there exists $y \in \mathbb{Q}$ so that $2y = x$] TRUE

is there a solution to $2y = x$ $y \in \mathbb{Q}$?

$y = x/2 \in \mathbb{Q}$

For all $\varepsilon \in \mathbb{R}$, $\varepsilon > 0$, there is a $\delta \in \mathbb{R}$, $\delta > 0$
so that [if $|x| < \delta$ then $x^2 < \varepsilon$]

$P(\varepsilon, \delta)$: if $x \in \mathbb{R}$, and $|x| < \delta$ ~~then~~ $x^2 < \varepsilon$

$\forall \varepsilon, \varepsilon \in \mathbb{R}, \varepsilon > 0, \forall \delta, \delta \in \mathbb{R}, \delta > 0, P(\varepsilon, \delta)$

$\exists, \forall$

- There exists $x \in A$ so that for all $y \in B$ we have $P(x, y)$.

$\to$ There exists $x \in \mathbb{N}$ so that [for all $y \in \mathbb{N}$ we have $xy > 1$.] TRUE

There exists $x \in \mathbb{Q}$ so that for all $y \in \mathbb{Q}$ we have $xy < y$.

Find any x / is there so that [for all $y \in \mathbb{N}$, $xy > 1$]

yes! $x = 2$

For all $y \in \mathbb{N}$, $2y > 1$,

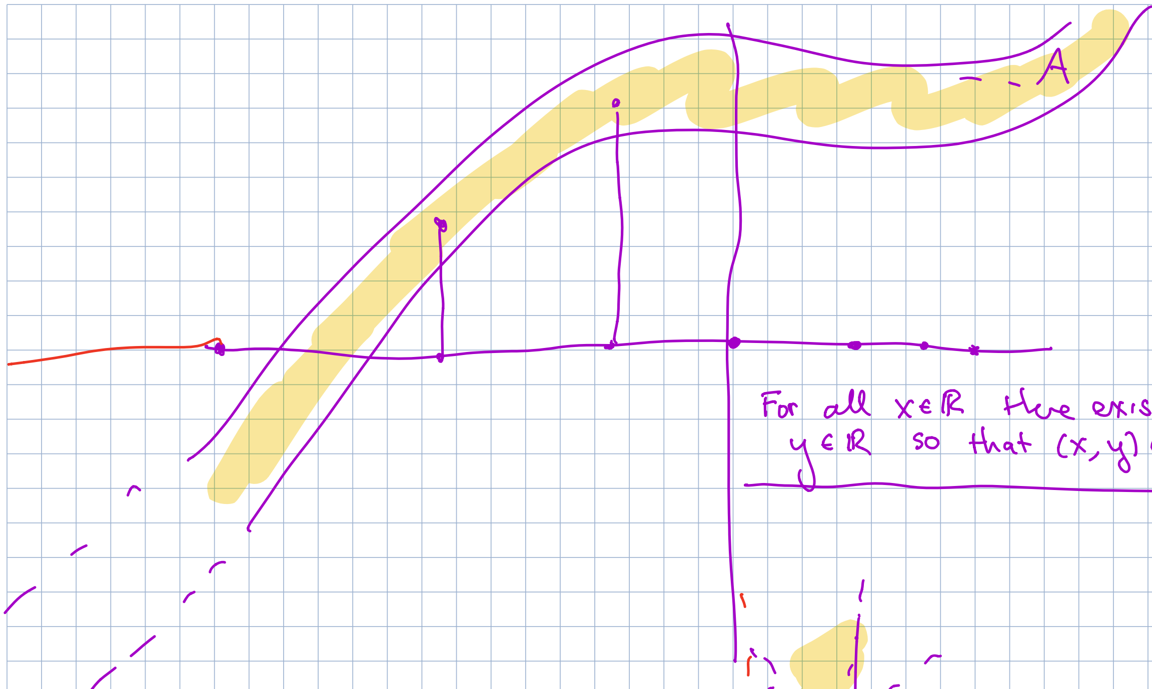
[For all $y \in \mathbb{Q}$ we have $xy < y$]

$x = 0$? No $y > 0$ $xy = 0 < y$ but if $y < 0$ then $xy = 0 \nless y$

$x > 0$ $xy > y$ if $y > 0$

$x < 0$ $xy > y$ if $y < 0$.

so this statement is False



For all $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ so that $(x, y) \in A$.

There exists $x \in \mathbb{R}$ so that for all $y \in \mathbb{R}$ we have $(x, y) \in A$.