

Truth Tables

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Compound statements can be complicated and Truth Tables let you calculate with them.

An example

Professor says: If you get an A on the final, or you get at least 90 on the homework, then you pass this course.

This statement is TRUE provided that the Professor told the truth (didn't lie) – whether or not you get an A in the course.

Analysis

- ▶ You get an A in this course (P)
- ▶ You get an A on the final (Q)
- ▶ You get at least 90 on the homework (R)

The promise is:

If (Q or R) then P.

How many possibilities?

Truth Table

$A \ B \ A \Rightarrow B$
 T T T
 T F F
 F T T
 F F F

$$(Q \vee R) \Rightarrow P$$

then \swarrow
 \uparrow OR

A on final
 Don't know
 A in course

Q	R	P	$Q \vee R$	$Q \vee R \Rightarrow P$
T	T	T	F	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Assign T or F to each statement

Count sequences

T, T, F or more
 generally sequences
 of length 3 with
 T or F.

Sequence is an element of

$$S = \{T, F\} \times \{T, F\} \times \{T, F\}$$

$$|S| = 2 \cdot 2 \cdot 2 = 8$$

Another example (see the text, Ch2.5)

Let P and Q be any statements. $(P \vee Q) \wedge \sim (P \wedge Q)$ reads as:

$(P \text{ OR } Q)$ and NOT $(P \text{ AND } Q)$.

P, Q

P	Q	$P \vee Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge \sim(P \wedge Q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	F	F

AND

exclusive OR
True if P or Q true
but not both.

Example

$$P \iff (Q \vee R)$$

- ▶ $xy = 0$ if and only if $x = 0$ or $y = 0$. — open sentence
- ▶ You will pass this course if and only if you get an A on the final or at least 90 on the homework.

P	Q	R	$Q \vee R$	$P \iff (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	F	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	T

$xy = 0$
 $x \neq 0$
 $y \neq 0$

here we
 no x, y
 in \mathbb{Z} or \mathbb{R}
 where this
 happens

pass course
 didn't get A
 didn't get 10

P	Q	$P \iff Q$
T	T	T
F	T	F
T	F	F
F	F	T

$$A = \{(x, y) \in \mathbb{R}^2 : xy = 0 \text{ and } x \neq 0 \text{ and } y \neq 0\}$$

$$A = \emptyset$$

Homework example

always false

Write a truth table for $(P \wedge \sim P) \vee Q$.

P	Q	$P \wedge \sim P$	$(P \wedge \sim P) \vee Q$
T	T	F	T
T	F	F	F
F	T	F	T
F	F	F	F