Biconditionals

The converse

If P then Q

Given an implication $P \implies Q$, its *converse* is the statement $Q \implies P$.

if Q then P

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Statement and Converse are different

If I own a BMW 335xi, then I own a car.

- P is "I own a BMW 335xi"
- Q is "I own a car"

The converse is "If I own a car, then I own a BMW 335xi".

$P \implies Q$ is true but $Q \implies P$ is false.

Biconditionals or Equivalence

$P \iff Q$ means "If P, then Q" AND "If Q, then P". It is often read "if and only if" since

- P if Q means $Q \implies P$
- P only if Q means $P \implies Q$.

It can also be read "necessary and sufficient" (P is necessary and sufficient for Q).

P is necessary and sufficient for Q

$P \Rightarrow Q$ is NOT THE SAME AS $Q \Rightarrow P$

$P \Leftrightarrow Q$ is the same as $Q \Leftrightarrow P$.

Truth Table for Equivalence

$P \Leftrightarrow Q$ P if and only if Q   P necessary and sufficient for Q

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$ means $P \Rightarrow Q$ AND $Q \Rightarrow P$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

P is logically equivalent to Q

Synonyms

- P if and only if Q
- P is necessary and sufficient for Q
- P is equivalent to Q
- If P, then Q, and conversely.

Sample problem

Put the statement "If $xy = 0$ then $x = 0$ or $y = 0$, and conversely" in the form "P if and only if Q".

P: $xy = 0$

Q: $x = 0$ OR $y = 0$.

$xy = 0$ if and only if ($x = 0$ OR $y = 0$).

If $xy = 0$ then $x = 0$ (FALSE) $x = 3$, $y = 0$

If $x = 0$ then $xy = 0$ (TRUE)