# And, Or, Not

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# And, Or, and Not

P : 18 is even
Q : 18 is a multiple of 3
R : 18 is a multiple of 5

Let $P$ and $Q$ be statements.

$P$ **and** $Q$ is a new statement that is True if **both** $P$ and $Q$ are True; and false otherwise.

P and Q true
18 is even and 18 is a multiple of 3

P and R False
18 is not a multiple of 5

$P$ **or** $Q$ is a new statement that is True if **either** $P$ or $Q$, or **both**, are True; and false otherwise.

P or R is true
R false and P true

P and Q true

**Not** $P$ is a new statement that is True if $P$ is False, and False if $P$ is $Q$.

Not P is the statement
18 is not even

P true      NOT P is False

Negation of P means the same as Not P.

# And

$P$ and $Q$ can be written $P \wedge Q$ (compare with set intersection).

$\wedge$ : and $\qquad$ $P \wedge Q$ means $P$ and $Q$

intersection $A \cap B$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$.

$x \in A$ is a statement

$(x \in A)$ and $(x \in B)$ could be written

$(x \in A) \wedge (x \in B)$

$A \cap B = \{x : (x \in A) \wedge (x \in B)\}$

# OR

$P$ or $Q$ can be written $P \vee Q$ (compare with set union)

P or Q $\qquad$ $P \vee Q$

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$= \{x : (x \in A) \vee (x \in B)\}$

# Not

Not $P$ can be written $\sim P$, or sometimes $\neg P$.

$\sim P$ $\qquad\qquad$ $\neg P$

$\overline{X} = \{x : x \notin X\}$ $\qquad$ ($X \subseteq U$ universal set).

$= \{x : \sim(x \in X)\}$

# Examples

Write the open sentences $x \neq y$ and $y \geq x$ as P and Q, P or Q, or not P.

$x \neq y$

$P(x, y)$ be the open sentence $x = y$.

$x \neq y$ is $\sim P(x, y)$.

$y \geq x$ means $y > x$ or $y = x$.

$P(x, y)$ is "$y > x$"

$Q(x, y)$ ~~means~~ is "$y = x$"

$y \geq x$ is "$P(x, y)$ or $Q(x, y)$".

# Example

Express the following in the form $P \wedge Q$, $P \vee Q$ or $\sim P$.

$$A \in \{X \in \mathcal{P}(\mathbb{N}) : |\overline{X}| < \infty\}$$

$|\overline{X}| < \infty$ :

$X = \{3, 4, 5, \ldots\} \subseteq \mathbb{N}$

$\overline{X} = \{1, 2\}$ finite

$Y = \{\text{even numbers}\}$ $|\overline{Y}| = \infty$

$\overline{Y} = \{\text{odd numbers}\}$

$|\overline{Y}| = \infty$

$A \in \mathcal{P}(\mathbb{N})$ meaning $A \subseteq \mathbb{N}$.

1. $A \subseteq \mathbb{N}$ $\quad P$

2. $|\overline{A}| < \infty$ : $\overline{A} = \mathbb{N} - A$ $\quad Q$

$|\mathbb{N} - A|$ is finite

Our statement is $P$ and $Q$ open sentence

# Truth Tables

Truth tables are an effective way to keep track of combinations of statements.

| P | Q | P and Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

| P | Q | P or Q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

| P | Not P |
|---|---|
| T | F |
| F | T |

"Formulas"