

# Deduction

# Syllogism ('modus ponens')

Suppose  $P$  and  $Q$  are statements, that  $P \implies Q$  is true, and that  $P$  is true. Then  $Q$  is true.

$P$  hypothesis statement  
 $Q$  conclusion statement

$$P \implies Q$$

statement

$P \implies Q$  is true

AND

$P$  is true

THEN

$Q$  is true



| $P$ | $Q$ | $P \implies Q$ |
|-----|-----|----------------|
| T   | T   | T              |
| T   | F   | F              |
| F   | T   | T              |
| F   | F   | T              |

# Indirect Proof

Suppose that  $P$  and  $Q$  are statements, that  $P \implies Q$  is true, and that  $Q$  is false. Then  $P$  is false.

$$\begin{array}{r} P \implies Q \\ \sim Q \\ \hline \sim P \end{array} \quad \begin{array}{l} \text{true} \\ \text{true (Q false)} \\ \text{true (P false)} \end{array}$$

| $P$ | $Q$ | $P \implies Q$ |
|-----|-----|----------------|
| T   | T   | T              |
| T   | F   | F              |
| F   | T   | T              |
| F   | F   | T              |

If  $X$  is a set with 3 elements then  $\wp(X)$  has 8 elements:

$P$ :  $X$  has 3 elements  
 $Q$ :  $\wp(X)$  has 8 elements

$P \implies Q$  true.

$X$  is a set,  $\wp(X)$  does not have 8 elements, then  $X$  does not have 3 elements

# Elimination

Suppose that  $P \vee Q$  is true, and  $P$  is false. Then  $Q$  is true.

$P$ : I live on the moon

$Q$ : I love math

$P$  OR  $Q$  is TRUE

$P$  is FALSE

MUST BE THE CASE THAT  
 $Q$  IS TRUE

| $P$ | $Q$ | $P \vee Q$ |
|-----|-----|------------|
| T   | T   | T          |
| T   | F   | T          |
| F   | T   | T          |
| F   | F   | F          |

~~FA~~

$P \Rightarrow Q$  true  
Q true

| P | Q | $P \Rightarrow Q$ |
|---|---|-------------------|
| T | T | T                 |
| T | F | F                 |
| F | T | T                 |
| F | F | T                 |