Negations

Negation examples

- x and y are both even, $(\sim (P(x) \wedge Q(\cancel{x}\,y)))$

x is even $P(x)$

y is even $Q(y)$

$P(x)$ AND $Q(y)$

NOT $(P(x)$ AND $Q(y))$

$\|$

$\sim P(x)$ OR $\sim Q(y)$

x is not even OR y is not even.

x or y is NOT even

$P(x) \wedge Q(y)$

$\sim (P(x) \wedge Q(y))$

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
F	F	F
F	T	T
T	F	T
T	T	T

More examples

① The square of every real number is non-negative. $(\overbrace{\forall x \in \mathbb{R}, x^2 \geq 0})$.

② There is an integer y so that $y^2 = 20$. $(\exists y \in \mathbb{Z}, y^2 = 20)$

Negation of #1:

TRUE

$\sim (\forall x \in \mathbb{R}, x^2 \geq 0)$ is $\overbrace{(\exists x \in \mathbb{R}, x^2 < 0)}^{\text{FALSE}}$

There is an $x \in \mathbb{R}$ so that $x^2 < 0$.

False ~~FALSE~~

② $\sim (\exists y \in \mathbb{Z}, y^2 = 20)$ is $(\forall y \in \mathbb{Z}, y^2 \neq 20)$ TRUE

For all $y \in \mathbb{Z}$, $y^2 \neq 20$

There is no integer y so that $y^2 = 20$

The square of any integer y is never 20.

Still more

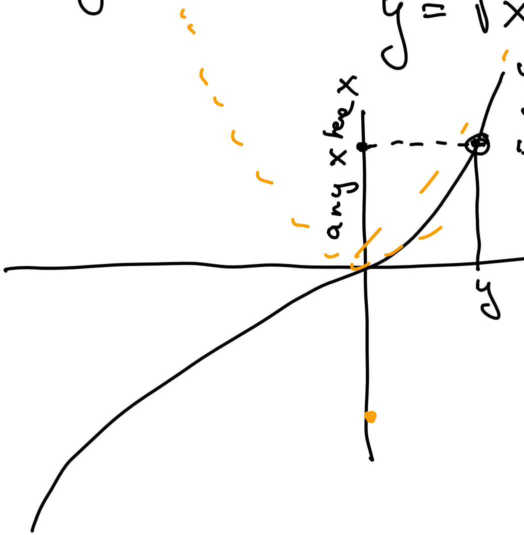
For every real number x there is a real number y so that $y^3 = x$.

$(\forall x, \exists y, y^3 = x)$

$P(x,y)$ $\quad y^3 = x$

TRUE if you can you solve the equation $y^3 = x$ with a real number y no matter what real number x is?

$y = \sqrt[3]{x}$ exists for any $x \in \mathbb{R}$.



$y^3 = x$

is there a y here? so $y^3 = x$ YES

NEGATION: There is an $x \in \mathbb{R}$ so that there does not exist $y \in \mathbb{R}$ with $y^3 = x$

There is an $x \in \mathbb{R}$ so that for all $y \in \mathbb{R}$, $y^3 \neq x$.

$\sim(\forall x, \exists y, y^3 = x) = (\exists x, \forall y, y^3 \neq x)$

Conditionals

- $P \implies Q$ is equivalent to $\sim P \vee Q$.
- $\sim(P \implies Q)$ is equivalent to $P \wedge \sim Q$.

If I own a car, then I am from South Dakota. $\simeq$ (Either I don't own a car or I am from South Dakota)

Negation: I own a car and I am not from South Dakota

$\forall$ people x, if x owns a car then x is from South Dakota.

$\sim$ ($\forall$ people, own car $\Rightarrow$ from south dakota)

$\exists$ person who owns a car who is NOT FROM SOUTH DAKOTA.

More examples

$f: \mathbb{R} \to \mathbb{R}$ a f^n
$b \in \mathbb{R}$

$\forall \varepsilon, \varepsilon > 0$ $\exists M \in \mathbb{Z}, M > 0$

For every positive real number ϵ, there is a positive integer M for which $x > M$ implies $|f(x) - b| < \epsilon$.

for all $x \in \mathbb{R}$ if $x > M$ then $|f(x) - b| < \varepsilon$. $P \Rightarrow Q$

Note implicit "for all x" in the implication.

Negation:

There is a positive real number ϵ so that for all positive integers M there is an $x > M$ and $|f(x) - b| \geq \epsilon$.

$P \wedge \sim Q$

$(\exists \varepsilon > 0)$ $(\forall M \in \mathbb{Z}, M > 0)$, $(\exists x \in \mathbb{R})$, so that $x > M$ and $|f(x) - b| \geq \varepsilon$.

$\forall \varepsilon, \varepsilon > 0$? $\exists \varepsilon, \varepsilon \leq 0$

$\forall \varepsilon \in \mathbb{R}_{>0}$