Logic and Statements

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Logic

- Informally, logic is the set of rules that govern reasoning.
- The rules of logic allow one to combine truths together to conclude other truths. For example, if we know that every bird has wings, and we know that a turkey is a bird, then we "automatically" know that a turkey has wings.
- Naively we might think that if we have a complete set of axioms, or basic truths, then using logic we could derive all other truths.
- The work of Godel showed that there are true statements that can't be proved. The book Godel, Escher, Bach by Douglas Hofstadter is a beautiful explanation of Godel's work that is accessible to everyone. See
  Hofstadter, Douglas R. (1999) [1979], Gödel, Escher, Bach: An Eternal Golden Braid, Basic Books, ISBN 0-465-02656-7,

Statements

A **statement** is a sentence which is either True or False. Some examples:

- Every buffalo is a mammal.
- Every system of n linear equations in n unknowns has a solution.
- There have been 62 presidents of the United States.
- There is an $x \in \mathbb{Q}$ such that $x^2 = 2$.

Non-statements

- Speak friend, and enter.
- $\{2x : x \in \mathbb{N}\}$.
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Naming statements and statements with variables

- P is the statement "Every odd number is prime."
- Q is the statement "No even number is prime."
- $P(x)$ is the statement: The integer x is even. The truth of this depends on x; this is really infinitely many statements, one for each integer x. When the truth depends on the values of the variables it is called an **open sentence.**

Some statements are mysterious

Book gives Goldbach Conjecture and Fermat's Last Theorem.

The Collatz Game: Pick a natural number x. If x is even, divide it by 2. If x is odd, multiply it by 3 and add 1. Repeat.

$3 \xrightarrow{3x+1} 10 \xrightarrow{\div 2} 5 \xrightarrow{3x+1} 16 \to 8 \to 4 \to 2 \to 1 \xrightarrow{3x+1} 4 \to 2 \to 1 \to$

$$7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 2, 1, \ldots$$

Let $C(x)$ be the statement:

if you start with x, you will eventually (after finitely many steps) reach the cycle $1, 2, 4, 1, 2, 4, \ldots$.

Is $C(x)$ always true? ← Is $C(x)$ true no matter what $x \in \mathbb{N}$ you start with?

Statement: $C(x)$ is true for any initial $x \in \mathbb{N}$. ???

Millenium Problems

$1,000,000 prize to solve any of them.

Poincare Conjecture solved

Rest of problems unknown.