Injective (1-1) and surjective (onto) functions

We introduce three fundamental properties that some functions have. These properties test your ability to work with quantifiers in a very fundamental way.

Injective functions

Definition (12.4 in the book): Let $f : A \to B$ be a function. Then

- f is called **injective** if, for all a, a' in A, if $a \neq a'$ then $f(a) \neq f(a')$. (Such f are also called "one-to-one" functions).

$\forall a \in A \, \forall a' \in A$, if $a \neq a'$ then $f(a) \neq f(a')$.

f is injective if, for all $a, a' \in A$,
If $f(a) = f(a')$ then $a = a'$!

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = 3x + 2$.

Prop: f is injective.

Proof: Let $a, a' \in \mathbb{R}$. Suppose $a \neq a'$.

Then $\left.\begin{aligned} f(a) &= 3a + 2 \\ f(a') &= 3a' + 2 \end{aligned}\right\}$ sufficient.

Proof: Let $a, a' \in \mathbb{R}$,

suppose $f(a) = 3a + 2 = f(a') = 3a' + 2$

$3a + 2 = 3a' + 2$

then $3a = 3a'$

and $a = a'$!

f is injective.

Surjective functions

$f: A \to B$

- f is called **surjective** if, for all $b \in B$, there exists $a \in A$ such that $f(a) = b$. (such f are also called "onto" functions.)

Note: whether a function is surjective depends on its codomain. It is always surjective onto its range.

$f: \mathbb{R} \to \mathbb{R}$

$f(x) = 3x+2$

We must show that: For all $b \in \mathbb{R}$, there exists an $a \in \mathbb{R}$ so that

$f(a) = b$

In other words

$f(a) = 3a+2 = b$

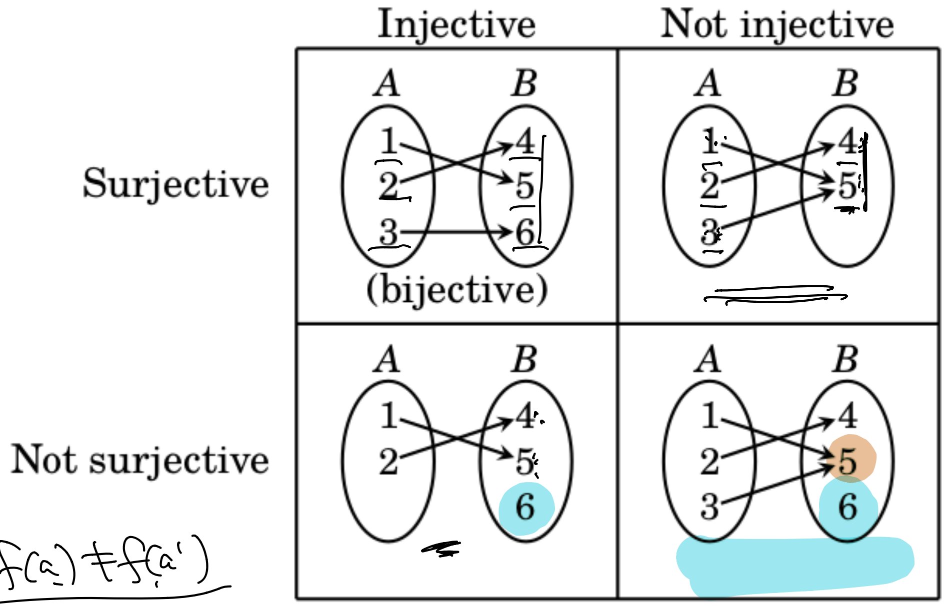
let $\cancel{b}$ $a = \dfrac{b-2}{3}$

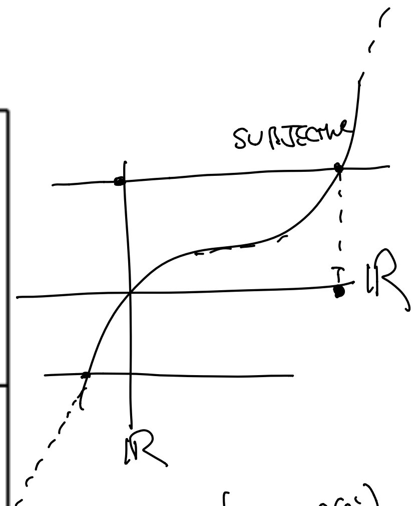
then $f(a) = 3\left(\frac{b-2}{3}\right)+2 = b$

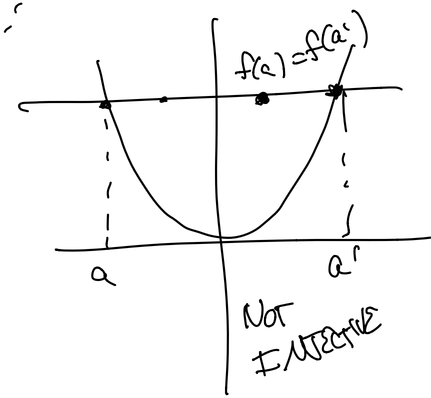
Therefore f is surjective.

Picture from page 229

BIJECTIVE = SURJECTIVE AND INJECTIVE.







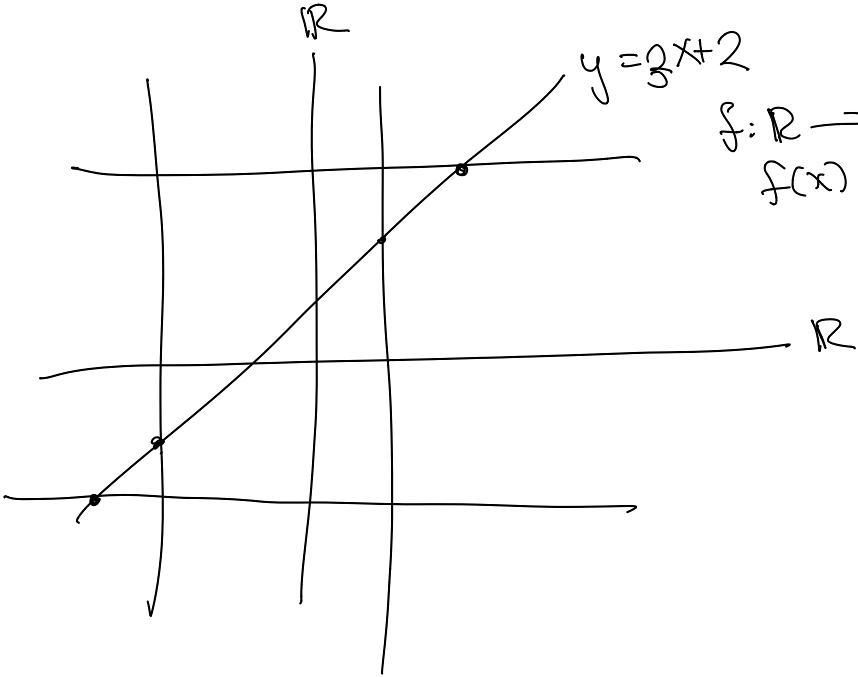
$a \neq a' \Rightarrow f(a) \neq f(a')$

injective

$\forall a \in$ codomain there exists b in the domain with $f(a) = b$. surjective

Bijective functions

- f is called **bijective** if it is both surjective and injective.



$f : \mathbb{R} \to \mathbb{R}$

$f(x) = 3x+2$

Injective functions