Selected Problems on Equivalence Relations

11.3.3

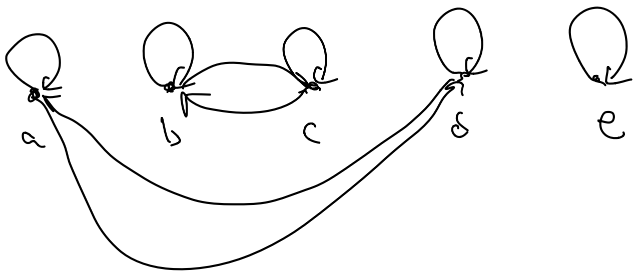
Let $A = \{a, b, c, d, e\}$. Suppose that R is an equivalence relation on A and R has three equivalence classes. Also aRd and bRc. Write out R as a set.

$R \subseteq A \times A$

1) R is reflexive $\{(a,a)\ (b,b)\ (c,c)\ (d,d)\ (e,e)\} \in R$

2) Know $(a,d) \in R$ so by symetry $(d,a) \in R$

$(b,c) \in R$ so " " $(c,b) \in R$



$[e] = \{e\}$

$[a] = [d] = \{a,d\}$

$[b] = [c] = \{b,c\}$

$R = \{(a,a), (b,b), (c,c), (d,d), (e,e), (a,d), (d,a), (b,c), (c,b)\}.$

11.3.7

Define a relation R on $\mathbb{Z}$ as xRy if $3x - 5y$ is even. Prove that R is an equivalence relation and describe the equivalence classes.

1) R is reflexive. is xRx? test: $xRx \Leftrightarrow 3x-5x$ is even.
but $3x-5x = -2x$, which is even
so R is reflexive.

2) R symmetric. check $xRy \Rightarrow yRx$. xRy means $3x-5y$ is even.
what about yRx? Is $3y-5x$ even?

$$(3x-5y) - (3y-5x) = 8x - 8y$$

$$\underset{\text{even}}{(3y-5x)} = \underset{\text{even}}{\underline{(3x-5y)}} + (8y-8x)$$

this is even

so

$$xRy \Rightarrow yRx$$

3) transitivity

Suppose $3x-5y$ is even and $3y-5z$ is even, so xRz.

$3x-5y = 2K$ and $3y-5z = 2S$

$3x = 2K+5y$ and $5z = 3y-2S$

$3x - 5z = 2K + 5y - 3y + 2S = \underline{2y+2S+2K}$

11.3.13

Suppose that R is an equivalence relation on a finite set A, and every equivalence class has the same cardinality m. Express $|R|$ in terms of m and $|A|$.

Choose $a \in A$.

$$[a] = \{ x \in R \mid aRx \text{ i.e. } (a,x) \in R \}.$$

$|[a]| = m$ then there are m elements x you can put so $(a,x) \in R$.

For each $a \in A$ there are m elements x so that $(a,x) \in R$.

$|A|$ choices for first element.

m choices for second element.

total # is $m|A| = |R|$.

$xRy \iff 3x-5y$ even

$x=0 \qquad [0] = \{y \mid 3(0)-5y \text{ even}\}$

$= \{y \mid 5y \text{ even}\}.$

$[0] = \{\text{even integers}\}.$

$[1] = \{y \mid 3-5y \text{ even}\}.$

$= \{\text{odd integers}\}.$

$xRy \iff x, y$ have same parity.

~~x even~~

$x \backslash y$	even	odd
even	xRy	$x \not R y$
odd	$x \not R y$	xRy