$F_n = F_{n-1} + F_{n-2}$ $\qquad F_1 = 1,\ F_2 = 1$

1, 1, 2, 3, 5, 8, 13, 21, 34, - - - -
odd odd even odd odd even, - - - - -

F_n is even if $3|n$.
F_n is odd if $3\nmid n$.

$3|n \Rightarrow F_n$ even.

F_n even $\Rightarrow 3|n$ but the contrapositive of this is
$3\nmid n \Rightarrow F_n$ odd.

$F_n = F_{n-1} + F_{n-2}$ $\qquad n \geq 4$

$F_{n-1} = F_{n-2} + F_{n-3}$ $\qquad n \geq 3$

$F_n = F_{n-2} + F_{n-3} + F_{n-2} = 2F_{n-2} + F_{n-3}$ $\qquad n \geq 4$

So F_n has same parity as F_{n-3} for all $n \geq 4$.

Inductive hypothesis: $n \geq 4$
for all $k < n$, F_k is even $\Leftrightarrow$ k is a multiple of 3

When $n = 4$, F_1, F_2, F_3 satisfy
F_k is even $\Leftrightarrow$ k is a multiple of 3
and that is true.

Assume true for $n \geq 4$.

We want to show F_n is even $\Leftrightarrow 3|n$.

We know F_n has same parity as F_{n-3}.

$n-3 < n$ our hypothesis says F_{n-3} is even if and only if $3|(n-3)$.

① if $3|(n-3)$ then $n = (n-3)+3$, 3 also divides n.
F_n is even since F_{n-3} is even.

② if $3\nmid(n-3)$, then $3\nmid n$.
and conversely.
So F_{n-3} is odd and so is F_n.