Prove: $F_1^2 + F_2^2 + F_3^2 + \dots + F_n^2 = F_n F_{n+1}$

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 2 & 3 & 5 & 8 \end{array}$$

$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 1+1+4+9+25 = 34+6 = 40 = F_5 \cdot F_6$

Proof by induction:

$$F_1^2 = 1^2 = F_1 F_2 = 1 \cdot 1$$

Suppose $F_1^2 + \dots + F_n^2 = F_n F_{n+1}$

Look at $F_1^2 + \dots + F_{n+1}^2 = (F_1^2 + \dots + F_n^2) + F_{n+1}^2$

$= F_n F_{n+1} + F_{n+1}^2$

$= F_{n+1}(F_n + F_{n+1})$

$= F_{n+1} F_{n+2}$

as desired.