Complement

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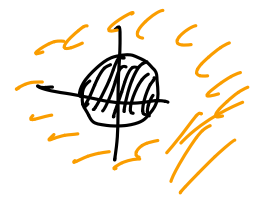
- The complement of a set is defined when our given set is understood to be a subset of some much larger set called the **universe** or **universal set.**
- When X is a set and its universal set U is specified (or understood) then the **complement** $\overline{X}$ is the set $U - X$.

Naive: $\overline{X} = \{x : x \notin X\}$

$X = \{4, 20, \text{banana}\} \subseteq U = \{x : x \text{ is a fruit or } x \in \mathbb{N}\}$

$\overline{X} = U - X = \{x : x \text{ a fruit (but not a banana) or a } x \in \mathbb{N} \text{ but not } 4 \text{ or } 20\}$

$X = \{(x,y) : (x,y) \in \mathbb{R}^2 \text{ and } x^2+y^2 \leq 1\}$



$\overline{X} = \{(x,y) : (x,y) \in \mathbb{R}^2 \text{ where } x^2+y^2 > 1\}$

$U = \mathbb{R}^2$

Example

- P is the set of prime numbers, with universal set $U = \mathbb{N}$. What is $\overline{P}$?

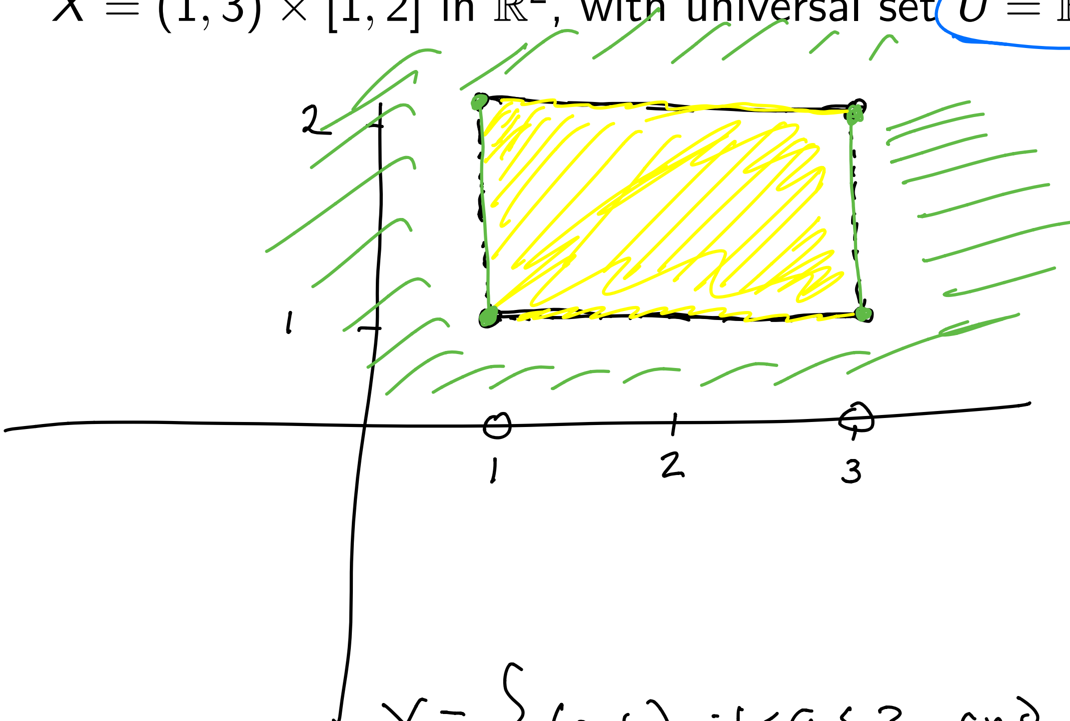
x is a prime number if it is a natural number greater than one whose only divisors are 1 and x

2, 3, 5, 7, 11, 13, 17, 19, ...

$$\overline{P} = U - P = \mathbb{N} - P = \{1, \underbrace{4, 6, 8, 9, 10, \ldots}_{\text{composite}}\}$$

Example

$X = (1,3) \times [1,2]$ in $\mathbb{R}^2$, with universal set $U = \mathbb{R}^2$. Sketch $\overline{X}$.



$$X = \left\{ (a,b) : \underset{a \in (1,3)}{1 < a < 3} \text{ and } \underset{b \in [1,2]}{1 \leq b \leq 2} \right\}$$

X is yellow

$\overline{X}$ is green

Example

Suppose:

- $A = \{x : x \in \mathbb{N}, x \text{ is even and } 0 \leq x \leq 8\}$ $A = \{2, 4, 6, 8\}$
- $B = \{x : x \in \mathbb{N}, x \text{ is odd and } 0 \leq x \leq 8\}$ $B = \{1, 3, 5, 7\}$
- $U = \{x : x \in \mathbb{N}, 0 \leq x \leq 8\}$. $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

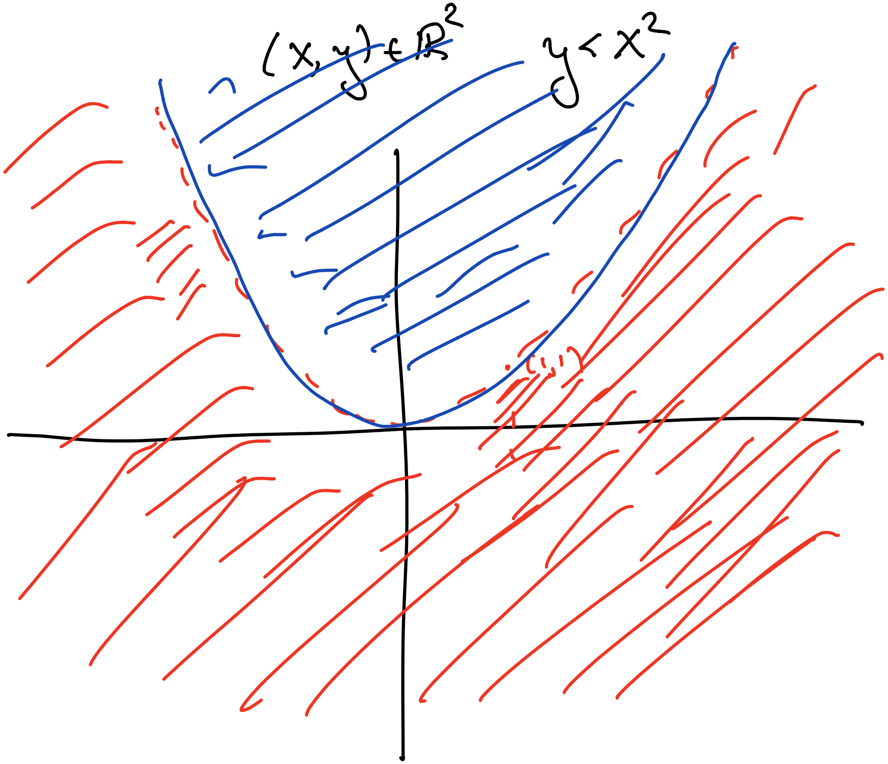
What is $\overline{A} \cap B$?

$$\overline{A} = U - A = \{1, 3, 5, 7\}$$

$$\overline{A} \cap B = \{1, 3, 5, 7\} = B$$

Example

$X = \{(x,y) \in \mathbb{R}^2 : y < x^2\}$ with universal set $\mathbb{R}^2$. Sketch $\overline{X}$.



$y = x^2$

X is red

$\overline{X}$ is purple.