Union, Intersection, and Difference of Sets

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Union

If A and B are sets, the union C of A and B, written $C = A \cup B$, is the set of elements of either A or B or both.

union is $\cup$

inclusive or

$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$

$$\{1, 2, 3\} \cup \{3, 5, 6\} = \{1, 2, 3, 5, 6\} = A \cup B$$

A B

Intersection

If A and B are sets, the intersection C of A and B, written $C = A \cap B$, is the set of elements in both A and B.

$\cap$

$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

$$\{1, 2, 3\} \cap \{3, 5, 6\} = \{3\}$$

$\{3\}$

Difference

If A and B are sets, the difference C of A and B, written $C = A - B$, is the set of elements in A but not in B.

$$A - B = \{x : x \in A \text{ and } x \notin B\}.$$

$$\{1, 2, 3\} - \{3, 5, 6\} = \{1, 2\} \quad = A - B$$

$B - A = \{x : x \in B \text{ but } x \notin A\}$

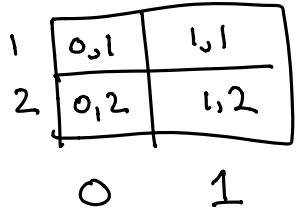
$= \{5, 6\}$

Example

$A = \{0, 1\}$ and $B = \{1, 2\}$. What is $(A \times B) \cap (B \times B)$?

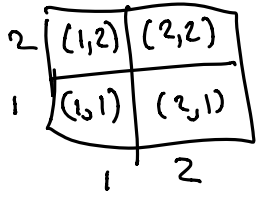
$(A \times B) \cap (B \times B)$

$A \times B$:



$\therefore \{(0,1), (1,1), (0,2), (1,2)\}$

$B \times B$:



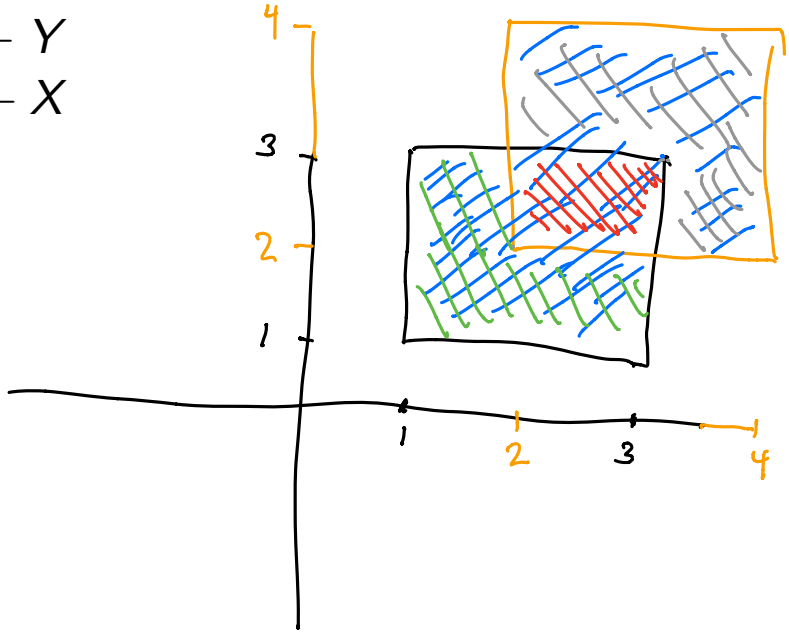
$\{(1,2), (2,2), (1,1), (2,1)\}$

$(A \times B) \cap (B \times B) = \{(1,2), (1,1)\}$

Example

Let $X = [1,3] \times [1,3]$ and $Y = [2,4] \times [2,4]$ in $\mathbb{R}^2$. Sketch the sets

- $X \cup Y$
- $X \cap Y$
- $X - Y$
- $Y - X$



X −

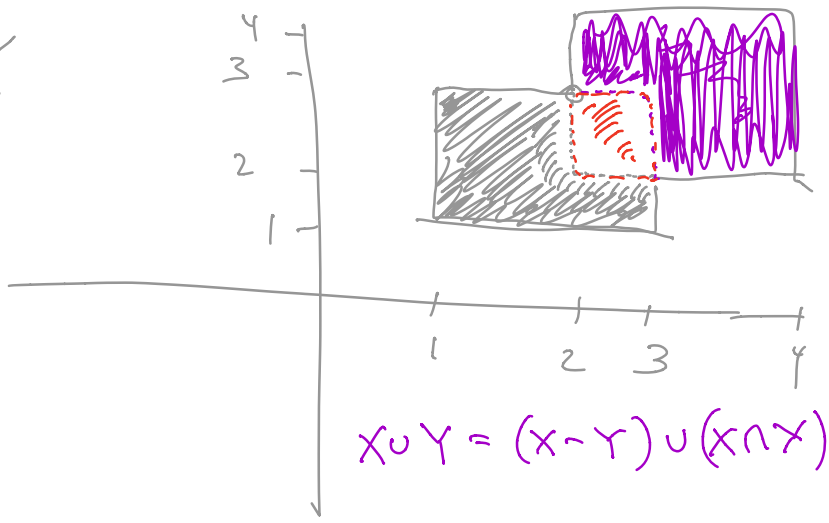
Y −

X ∪ Y −

X ∩ Y

X − Y

$X - Y$



$X - Y$

$Y - X$

$$X \cup Y = (X - Y) \cup (X \cap Y) \cup (Y - X)$$