

$$\underline{A} = \{X : \underline{X \text{ is a set and } X \notin X}\} \quad \text{Russell's trick}$$

$\{\{1,2\}, \{1,3\}\}$ example

$\mathbb{Z} \in A$ because $\mathbb{Z} \notin \mathbb{Z}$

$\emptyset \in A$ because $\emptyset \notin \emptyset$ $\mathbb{Z} = \{\dots, -3, -2, -1, 0, \dots\}$

⋮

Is $A \in A$?

If $A \in A$... hmmm... elements of A are those sets that aren't elements of A .

If $A \notin A$, ... hmmm. Then $A \in A$ because elements of A are exactly the sets that aren't elements of themselves.

PARADOX

Frege

Zermelo Frankel axioms of set theory.

Foundations of Mathematics