Final Exam
Math 2710, Fall 2019

Instructions: This exam comes in two parts, **A** with five problems and **B** with five problems, each worth 20 points. You may do any **four** problems from part A and any **four** problems from part B. I will count your best scores in each part.

Name:

A.1. (20 points) Let $A = \{0, 1, 2, 3, 4\}$ and $B = \{1, 3, 5\}$. Let f be the function

$$f \subset A \times B = \{(0, 1), (3, 3), (2, 3), (1, 1), (4, 1)\}$$

and let $g : B \to A$ be the function

$$g \subset B \times A = \{(1, 0), (3, 2), (5, 2)\}$$

Give $g \circ f$ as a subset of $A \times A$, explaining how you obtained your answer.

A.2. (20 points) Let $A = \{1, 2, 3, 4, 5\}$.

1. Give a function $f : A \to A$ that has $f(1) = 3$ and that is neither surjective nor injective, and justify your answer.
2. Give a function $g : A \to A$ that has $g(1) = 3$ and is bijective, and justify your answer.

A.3. (20 points) Let $f : \mathbb{R} - \{2\} \to \mathbb{R} - \{1\}$ be the function $f(x) = \frac{x+2}{x-2}$. Show that f is bijective by finding the inverse function $f^{-1} : \mathbb{R} - \{1\} \to \mathbb{R} - \{2\}$.

A.4. (20 points) Let S be the set of infinite sequences $a_1, a_2, \ldots$ with all a_i either 0 or 1. Explain how Cantor's diagonalization argument proves that S is uncountable.

A.5. (20 points) Prove that the set $\mathbb{N} \times \mathbb{N}$ is countable.

B.1. (20 points) Let P, Q, and S be propositions. Prove that P or (Q and S) is equivalent to (P or Q) and (P or S).

B.2. (20 points) Find the remainder when 5^{2020} is divided by 7.

B.3. (20 points) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by the formula $f(x, y) = 7x + 11y$. Prove that f is surjective.

B.4. (20 points) Prove that

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n} = 2 - \left(\frac{n+2}{2^n}\right)$$

for all $n \geq 1$.

B.5. (20 points) Let $a(n)$ be the sequence $a(n) = \frac{2n}{n+1}$. Prove (using the definition of limit of a sequence) that the limit as $a(n)$ is 2.